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CONTENTS

- O. POSTHUMUS: "Some remarks concerning the Palaeozoic Flora of Djambi, Sumatra". (Communicated by Prof. J. C. SCHOUTE), p. 628.
- F. A. H. SCHREINEMAKERS: "Equilibria in systems, in which phases, separated by a semi-permeable membrane" XXIII, p. 635.
- C. H. H. SPRONCK, K. HOEFNAGEL, W. HAMBURGER and A. J. BOEKELMAN: "Concerning the Transmutant of the Tubercle-bacillus *BTTx* as Tuberculosis-diagnosticum", p. 643.
- G. SCHAAKE: "The Quadratic Birational Correspondence between Two Linear Complexes of Rays". (Communicated by Prof. HENDRIK DE VRIES), p. 657.
- J. P. WIBAUT and G. L. C. LA BASTIDE: "On some derivatives of di-(2-pyridyl)-amine, and on tri-(2-pyridyl)-amine". (Communicated by Prof. A. F. HOLLEMAN), p. 664.
- M. H. A. NEWMAN: "On the foundations of combinatory Analysis Situs; Additions and Corrections". (Communicated by Prof. L. E. J. BROUWER), p. 670.
- S. R. DE BOER: "The Influence of Temperature on the Respiration of *Phycomyces Blakesleeanus* Burgeff". (Communicated by Prof. F. A. F. C. WENT), p. 674.
- ADRIAAN VAN MAANEN: "The proper motions of the globular clusters Messier 13, 56 and 2 and their internal motions", p. 681.
- A. J. P. VAN DEN BROEK: "On the continuance of the Increase of Stature in Holland". (Communicated by Prof. L. BOLK), p. 685. (With three maps).
- E. HORNE CRAIGIE and RICHARD M. BRICKNER: "Structural parallelism in the midbrain and tweenbrain of teleosts and of birds". (Communicated by Dr. C. U. ARIËNS KAPPERS), p. 695.
- K. MENDER: "Zusammenhangsstufen und Cantorsche Mannigfaltigkeiten". (Communicated by Prof. L. E. J. BROUWER), p. 705.
- K. MENDER: "Bemerkungen zur zweiten Untersuchung über allgemeine Metrik". (Communicated by Prof. L. E. J. BROUWER), p. 710.
- LAURETTA BENDER: "An Experimental Study of the Cerebellar Control of the Vocal Organs". (Communicated by Prof. G. VAN RIJNBEEK), p. 715. (With one plate).
- W. NIEUWENKAMP: "The Meteorite of Ellemeet (after that of Uden in 1840, and that of Blauwkapel in 1843, the third in the Netherlands)". (Communicated by Prof. L. RUTTEN), p. 724.

*Paleobotany. — Some remarks concerning the Palaeozoic Flora of Djambi, Sumatra.* By O. POSTHUMUS. (Communicated by Prof. J. C. SCHOUTE.)

(Communicated at the meeting of April 30, 1927).

During the geological exploration of the residency Djambi by Dr. A. TOBLER and his collaborators, Ir. GÖLLNER discovered remains of fossil land-plants in palaeozoic strata at Moeara Ketidoeran Siamang, near the Soengei (river) Merangin in the district Bangko in the interior of the residency. As this part of Sumatra is situated about midway between British India and Australia, where a well-developed *Glossopteris* flora is found, one should expect to find plants of this type here too; especially as similar forms have been recorded from Serawak<sup>1</sup>). The fossil plants from Djambi, however, were more similar to those of Western Europe. The specimens collected at that occasion were first mentioned by JONGMANS in the report on the geological explorations in Djambi<sup>2</sup>), and in 1925 a detailed description of them was published by JONGMANS and GOTHAN<sup>3</sup>).

Because of the great importance of this locality for the study of the relationship between the *Glossopteris* flora and that of the Arctocarbonic type, further collecting was planned. Some scientific societies and private persons, a few years ago, put funds at the disposal of Dr. JONGMANS, who, however, was prevented by circumstances to make the voyage. In 1925 these funds were transmitted to me and during an expedition into the interior of Djambi, which was made in combination with a geological re-investigation by the Geological Survey of the Netherland East Indies, a collection of these fossil plants was made. A preliminary report has already been published<sup>4</sup>).

I hoped to have occasion to study at least part of the material, which I had collected, as a compensation for the time and trouble, necessary for the collecting of the materials. Instead of going to Java directly, I returned to the Netherlands with the intention to work out the results, and though the specimens had already arrived in Holland long ago and I often requested to have put them at my disposal for study, I never saw back any of the fossils, which are now in the hands of Dr. JONGMANS.

The following remarks are based upon some observations during the

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<sup>1</sup>) TENNISON—WOODS, 1885, p. 583.

<sup>2</sup>) JONGMANS in TOBLER, 1923, p. 193, 308, 541.

<sup>3</sup>) JONGMANS und GOTHAN 1925.

<sup>4</sup>) ZWIERZYCKI and POSTHUMUS 1926.

collecting-work, and on some notes and sketches, then made. They are to be considered only as an addition to the preliminary report; the publication by JONGMANS and GOTHAN is now also quoted; it was then not yet known to me.

The species, which have been found, are given in the following list :

	Djambi		Stéph.			Perm.		OA
	J	P	Ct	Bc	Py	B	P	
1. Pecopteris Verbeeki J. et G. . . . .	—							
2. P. polymorpha Brongn. . . . .	—		—	—	—	—	—	
3. P. hemitelioides Brongn. . . . .	—		—	—	—	—	—	
4. P. arborescens Brongn. . . . .	—		—	—			—	
5. P. cf. Miltoni Brongn. . . . .		—						—
6. Cyclopteris spec. . . . .	—							
7. Callipteridium cf. gigas (Schlotheim) Weiss . .		—	—	—	—	—	—	
8. Taeniopteris multinervis Weiss . . . . .		—				—	—	—
9. Gigantopteris americana White . . . . .		—					—	
10. Calamites Suckowi Brongn. . . . .	—	—	—	—		—	—	—
11. Sigillaria Brardi Brongn. . . . .		—	—	—	—	—	— <sup>1)</sup>	
12. Lepidodendron cf. Gaudryi B. Ren. . . . .		—	—				—	—
13. L. felis oculus (Abbado) Zeller . . . . .		—						—
14. Stigmaria ficoides Brongn. . . . .	—	—	—	—		—		— <sup>2)</sup>
15. Sphenophyllum oblongifolium Germar . . . .	—	—	—	—	—	—	—	—
16. Cordaites borassifolius Stbg. . . . .	—	—		—				
17. Tobleria bicuspis J. et G. . . . .	—							
18. Dadoxylon spec. . . . .	—	—						

Abbreviations :

J = JONGMANS and GOTHAN 1925.	Py = Basin of Perrecy.
P = Observation of the author.	B = Basin of Bert.
Ct = Basin of Commentry.	P = Other Permian strata.
Bc = Basin of Blanz and Creusot.	OA = Upper Palaeozoic strata of East-Asia.

On these specimens the following remarks may be made :

1—5. The fronds of Pecopteris belong to a group, which is known from

<sup>1)</sup> RENAULT 1896, p. 193.

<sup>2)</sup> GOTHAN 1915, p. 268.



the Stephanian and Permian of Europe, and also from China and Malakka<sup>1)</sup>).

8. The specimens of *Taeniopteris* from Djambi probably belong to several species.

9. Some of the specimens show a distinct nervation.

11. Several specimens with the leaves still adhering were found at the Moeara Ketidoeran Siamang.

12. The specimens much resemble the fossil, figured by JONGMANS and GOTHAN on pl. IV, fig. 5. It does not seem probable that the twigs, which are mentioned in the preliminary report as *Walchia* spec.<sup>2)</sup> belong to the same plant.

13. From this species some specimens were found at the Soengei Maroes.

14. This species is rather frequent in some localities; some basal stumps of the stem in situ showed by their position that the strata, in which they occurred, were not turned upside down.

The strata, which contain the fossil plants, belong to a series of concordant layers in which banks of vulcanic material and other ones containing marine fossils were intercalated. On some places they had the character of a *Stigmaria*-bank, covered by a bed of lava; on other places they were developed as a shale, containing much *Cordaite*s, or as a rather coarse sandstone with much impressions of *Cordaite*s, *Taeniopteris* and *Pecopteris*; the locality at the Moeara Ketidoeran Siamang was especially interesting, because the plants were found imbedded in a white clay, which was still plastic; numerous leaves and seeds of *Cordaite*s, leaves of *Pecopteris*, *Taeniopteris* and leaves and stems of *Sigillaria* occurred in it.

When collecting the fossils, I did not get the impression, that the flora at the different localities was much different. The marine fossils, which formerly were considered to be of Permian age, occurred in strata, belonging to the same series as the plant-bearing strata and occurring at different height, sometimes alternating with the latter; hence both kinds may be considered to be of the same age.

When discussing the age of strata, situated at the boundary of two formations, i.e. the Carboniferous and Permian, one should bear in mind that the criteria, used by the authors, may differ largely.

The olders of the series quoted here for comparison, that of Commentry, was by RENAULT and ZEILLER<sup>3)</sup>, later by ZEILLER<sup>4)</sup>, taken to be of Carboniferous age; but STERZEL<sup>5)</sup> included it in the Permian formation, because of a number of species, which are known elsewhere from Lower Permian strata only. Though most species may be found both in upper Carboniferous and in Permian beds, some species of *Sigillaria* and *Sphen-*

<sup>1)</sup> EDWARDS 1926, p. 171.

<sup>2)</sup> POSTHUMUS 1926, p. 215.

<sup>3)</sup> RENAULT et ZEILLER 1890, p. 713.

<sup>4)</sup> ZEILLER 1894, p. 275.

<sup>5)</sup> STERZEL 1895, p. 340.

teris, which are characteristic for deposits of the upper Carboniferous, were wholly absent.

In KAYSER, *Lehrbuch der Formationskunde*, these strata are mentioned to be of Permian age; in the French handbook of de LAPPARENT and HAUG, they are included in the upper Carboniferous.

From the East Asiatic regions T. G. HALLE<sup>1)</sup> draws about the same distinction as STERZEL: The upper Shibotse series is, according to him, doubtless of Permian age, as indicated by the occurrence of *Sphenophyllum Thoni* "one of the most distinctive species of the Permian". This species also occurs in the basin of Commentry<sup>2)</sup>.

ZALESSKY<sup>3)</sup> however thinks this species not be decisive of Permian age; his opinion agrees with that of the French palaeobotanists: the beginning of the permian is indicated by the appearance of *Callipteris conferta*, *Taeniopteris multinervis* and *Walchia filiciformis*. The first species, however, already occurs in the beds of Perrecy<sup>4)</sup>, which are considered by ZEILLER to be of Carboniferous age. Those of Bert are mentioned by all authors as belonging to the Permian formation<sup>5)</sup>, because of the occurrence of *Taeniopteris multinervis*, *Walchia linaeriaefolis*, and the abundance of *Callipteris conferta*. Most of the other species occur in Permian and upper Carboniferous strata.

The same difficulty arises for the localities of Chan-Si; ZEILLER<sup>6)</sup> mentions the occurrence of *Stigmaria ficoides*, *Lepidodendron* cf. *Gaudryi* from there, together with *Taeniopteris multinervis*. The first two species may be used as an argument for Stephanian age; the latter for Permian age, because, according to ZEILLER, it is found elsewhere in Permian strata only. But as a criterion for the age this argument is of relative value only: GOTHAN<sup>7)</sup> thinks these grounds to be insufficient for considering these strata to be of Permian age.

The localities in Djambi are mentioned by JONGMANS and GOTHAN<sup>8)</sup> as being of Carboniferous age, 10, because *Stigmaria* and *Lepidodendron* are found, 20, because species characteristic of the Permian do not occur. Of *Stigmaria* good specimens are found in Djambi. In the Saarbassin and in Thuringia they do not occur in the lower Permian (*Rotliegendes*); from Bert, however, which is considered to be of Permian age by all authors, its occurrence is mentioned by GRAND'EURY<sup>9)</sup> and ZEILLER<sup>10)</sup>, and from

1) HALLE 1924, p. 23.

2) RENAULT et ZEILLER 1888, p. 488, pl. 52, fig. 10.

3) ZALESSKY 1907, p. 523.

4) ZEILLER 1906, p. 232.

5) ZEILLER 1906, p. 251.

6) ZEILLER 1901, p. 451.

7) GOTHAN 1915, p. 267.

8) JONGMANS und GOTHAN 1925, p. 296.

9) GRAND'EURY 1877, p. 519.

10) ZEILLER 1906, p. 181.



East Asia by ZEILLER<sup>1)</sup>, where it is found in association with *Taeniopteris multinervis*. If the presence of *Stigmaria* is used as a criterion for Carboniferous age, as is done by GOTHAN, then *Taeniopteris multinervis* may be found in the Carboniferous too. In this respect GOTHAN draws the boundary lower than the other authors.

From Southern Yun-Nan, ZEILLER<sup>2)</sup>, mentions *Stigmaria* from the same locality as *Gigantopteris nicotiaenifolia*, from strata which are doubtless of Permian age. He also mentions the occurrence of similar forms from the Triassic of Baccarat, described by FLICHE<sup>3)</sup> as *Stigmarites*, but which, according to personal observations by ZEILLER<sup>4)</sup>, differ in nothing from *Stigmaria*. Moreover the occurrence of *Stigmaria*, may be called rather probable in the strata which contain *Sigillaria*, as is the case in the Permian formation.

As well as the decisive value of the occurrence of *Taeniopteris multinervis* and *Callipteris conferta*, which even by ZALESSKY are considered to occur in Permian strata only, can be doubted, as has been done by GOTHAN, the same may be the case with that of *Stigmaria* for Carboniferous strata, though GOTHAN considers it to be a rather decisive point.

The only remaining argument against Permian age is the occurrence of the genus *Lepidodendron*. But *L. felis oculis*, which is elsewhere known from the above mentioned East Asian strata only, must be left out of consideration. And also the other specimens, which much resembles *L. Gaudryi* from Commentry and from the East-Asian localities, originates from these transitional strata; as long as no more localities are known, it should be used as an argument against Permian age with at least the same reserve as GOTHAN<sup>5)</sup> expresses for *L. Schmalhauseni* Zal and *L. kirgisca* Zal.<sup>6)</sup>

If the list, given above, on the preliminary character of which stress should be laid, is considered, it becomes obvious that *Pecopteris Verbeeki* J. et G., *Tobleria bicuspis* J. et G., *Dadoxylon spec.* and *Cyclopteris spec.* are of no value for determining the age. The same is the case with the specimens mentioned as *Pecopteris cf. Miltoni*, as long as they have not been investigated more carefully; but it is interesting to see, that SCHENK<sup>7)</sup> mentions similar forms from Lui-pa-kon in Hu-nan from strata which certainly are not older than those from Djambi. From the remaining forms the three species of *Pecopteris*, *Callipteridium gigas*, *Taeniopteris multinervis*, *Gigantopteris americana*, *Sigillaria Brardi*, *Calamites Suckowil* and *Sphenophyllum oblongifolium* have been found in Permian strata;

1) ZEILLER 1901, p. 438, pl. 7, fig. 8.

2) ZEILLER 1907, p. 492, pl. 14, fig. 18.

3) FLICHE 1906, p. 138, pl. 13, fig. 2.

4) ZEILLER 1907, p. 493.

5) JONGMANS und GOTHAN 1925, p. 298.

6) ZALESSKY 1918, p. 49, pl. 4, fig. 5, 5a; p. 53, pl. 12, fig. 5.

7) SCHENK 1883, p. 230.

*Lepidodendron felis oculis*, L. Caudryi together with *Taeniopteris multinervis* in strata which according to ZEILLER<sup>1)</sup>, are of permo-carboniferous age. The only possible argument against Permian age, except the occurrence of *Stigmaria ficoides*, which has been discussed above, is the occurrence of *Cordaite borassifolius*, which has not yet been mentioned from the Permian. The delimitation of the "species" of *Cordaite*, however, is rather difficult and analogous forms are known from the Permian too.

As a species, characteristic for Permian strata, only *Taeniopteris multinervis* and *Gigantopteris americana*<sup>2)</sup> can be mentioned.

As far as data are available, they indicate that the fossil flora of Djambi shows most resemblance with the *Gigantopteris* flora of Eastern Asia and especially with the lowest part of it. Even if the boundary between Carboniferous and Permian formation is traced as high as possible, even then some arguments are left for a Permian age. Therefore it seems reasonable to indicate these strata as lower Permian or eventually as the transition, carboniferous-permian, as has already been done<sup>3)</sup>. For a more detailed opinion one should also take in account the results of the study of the marine fossils, which occur intercalated between the plant bearing strata.

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**Chemistry.** — *Equilibria in systems, in which phases, separated by a semipermeable membrane.* XXIII. By Prof. F. A. H. SCHREINEMAKERS.

(Communicated at the meeting of June 25, 1927).

*Influence of the temperature and the pressure on osmotic systems.*

We take the osmotic equilibrium:

$$E_1 \mid E_2 \dots \dots \dots (1)$$

in which we keep constant the total composition of each of both the separate systems at the left and at the right of the membrane. In order to find the number of licences of this equilibrium, we can use a.o. the membrane-phase-rule. (Comm. VII and VIII) viz.:

an osmotic equilibrium, which contains at the one side of the membrane  $n_1$  components in  $r_1$  phases and at the other side  $n_2$  components in  $r_2$  phases, has, when there are  $d$  diffusing substances:

$$n_1 + n_2 - (r_1 + r_2) + 3 - d \dots \dots \dots (2)$$

licences. If there is one diffusing substance only, as in the cases which we considered till now, then the osmotic equilibrium has, therefore:

$$n_1 + n_2 - (r_1 + r_2) + 3 - 1 = (n_1 - r_1 + 2) + (n_2 - r_2 + 2) - 2 \quad (3)$$

licences. Consequently we can say:

the number of licences of an osmotic equilibrium with one diffusing substance is equal to the sum of the licences of both the separate systems, diminished with two.

If we keep constant, as in (1), the total composition of each of the separate systems, then each has two licences, viz. the temperature and the pressure; consequently the osmotic equilibrium (1) has  $2 + 2 - 2 = 2$  licences.

We now take the osmotic equilibrium:

$$[(E_1)_{P_1} \mid (E_2)_{P_2}]_T \dots \dots \dots (4)$$

in which at the temperature  $T$  a pressure  $P_1$  is at the left of the membrane and a pressure  $P_2$  at the right. Of the three variables, viz.  $P_1$ ,  $P_2$  and  $T$  we may take two arbitrarily, therefore. If we change the temperature with  $dT$  and the pressures at the left and the right of the membrane with  $dP_1$  and  $dP_2$  then (4) passes into the osmotic system:

$$[(E_1)_{P_1+dP_1} \mid (E_2)_{P_2+dP_2}]_{T+dT} \dots \dots \dots (5)$$

Then the O.W.A. at the left and at the right side of the membrane has increased with:

$$d\xi_1 = \Delta H_1 \cdot dT - \Delta V_1 \cdot dP_1 \quad d\xi_2 = \Delta H_2 \cdot dT - \Delta V_2 \cdot dP_2. \quad (6)$$

in which  $\Delta H_1$  and  $\Delta V_1$  represent the osmotic increase of entropy and volume of the system  $E_1$ ;  $\Delta H_2$  and  $\Delta V_2$  represent the same for the system  $E_2$ .

In order to keep equal the O.W.A. on both sides of the membrane, we must satisfy, according to (6):

$$(\Delta H_1 - \Delta H_2) dT = \Delta V_1 \cdot dP_1 - \Delta V_2 \cdot dP_2 \quad \dots \quad (7)$$

from which appears, in accordance with above, that we may take arbitrarily two of the three increments.

With the aid of (7) we are able to deduce different properties. Previously we have deduced a.o. that we can change the pressure on both sides of the membrane in an osmotic equilibrium at constant temperature; those changes in pressure  $dP_1$  and  $dP_2$  must satisfy then formula (5) of communication XX, viz.:

$$\Delta V_1 \cdot dP_1 = \Delta V_2 \cdot dP_2 \quad \dots \quad (8)$$

which also follows at once from (7) if we put herein  $dT = 0$ .

As special case of (4) we may also take the osmotic equilibrium:

$$(E_1 \mid E_2)_{P, T} \quad \dots \quad (9)$$

in which the pressure is the same on both sides of the membrane; as by this the number of licences decreases with one, (9) therefore, has instead of two, one licence only. Consequently to each temperature  $T$  belongs a definite pressure  $P$  of the osmotic equilibrium; we are able to represent, therefore, the connection between temperature and pressure in a  $PT$ -diagram by a curve, which we shall call „the  $PT$ -curve of the osmotic equilibrium”; in fig. 1 such a curve is represented by  $aua_1$ . An arbitrary point  $u$  of this curve indicates, therefore, that at the temperature  $T_u = Ou_1$  the pressure on both sides of the membrane must be  $P_u = u_1 u$  and reversally, that we must take a temperature  $T_u = Ou_1$  when the pressure on both sides of the membrane is  $P_u = u_1 u$ .

The direction of this  $PT$ -curve follows from (7); if we put viz.  $dP_1 = dP_2 = dP$  then we find:

$$\frac{dT}{dT} = \frac{\Delta H_1 - \Delta H_2}{\Delta V_1 - \Delta V_2} = \frac{\Delta H_2 - \Delta H_1}{\Delta V_2 - \Delta V_1} \quad \dots \quad (10)$$

If we represent the heat of diffusion and the osmotic increase of volume at the transition of a little water from system  $E_2$  towards  $E_1$  (or from  $E_1$  towards  $E_2$ ) by  $Q_{2,1}$  and  $V_{2,1}$  (or  $Q_{1,2}$  and  $V_{1,2}$ ), then (10) passes into:

$$T \frac{dP}{dT} = \frac{Q_{2,1}}{V_{2,1}} = \frac{Q_{1,2}}{V_{1,2}} \quad \dots \quad (11)$$

which formula has the same form as that of CLAPEYRON; however, the magnitudes occurring in the second part have another meaning.

It numerator and denominator in (10) or (11) have the same sign, then the pressure of the osmotic equilibrium increases with increase of temperature; then the  $PT$ -curve is a curve, ascending with the temperature, as curve  $aa_1$  in fig. 1 or branch  $aub$  in fig. 2. If numerator and denominator have opposite sign, then the pressure decreases with increase of  $T$ ; then the  $PT$ -curve is a curve, descending with the temperature, as branch  $a_1vb$  in fig. 2. If the numerator becomes zero, then the pressure is maximum or minimum; if the denominator becomes zero, then the temperature is maximum or minimum. In fig. 2 the temperature is a maximum in  $b$ ; consequently the osmotic equilibrium (9) cannot exist above this temperature  $T_b$ . If we heat it notwithstanding above this temperature  $T_b$ , then, therefore, water must diffuse in any direction and another osmotic equilibrium must be formed; we shall refer to this later.

In order to apply our considerations to a definite case we take instead of (9) the osmotic equilibrium:

$$[Y+L \mid L_s]_{P,T} \dots \dots \dots (12)$$

in which at the left side of the membrane the binary system  $Y+L$  and on the right side a ternary liquid  $L_s$ . If we assume that fig. 1, III is valid for a definite  $P$  and  $T$  of (12), then the liquid at the left of the membrane has the composition  $w$ ; the liquid  $s$  at the right of the membrane is represented then by a point of the isotonic curve  $wm$  (fig. 1, III). If we represent the O.W.A., the osmotic increase of entropy and volume of the left system by  $\xi$ ,  $\Delta H$  and  $\Delta V$  and those of the liquid  $s$  by  $\xi_s$ ,  $\Delta H_s$  and  $\Delta V_s$ , then we have:

$$d\xi = \Delta H \cdot dT - \Delta V \cdot dP \quad d\xi_s = \Delta H_s \cdot dT - \Delta V_s \cdot dP \quad (13)$$

The direction of the  $P, T$ -curve of (12) is then defined by:

$$\frac{dP}{dT} = \frac{\Delta H - \Delta H_s}{\Delta V - \Delta V_s} \dots \dots \dots (14)$$

Of the many imaginable cases we shall discuss only some; for this we shall assume that  $\Delta V_s$ ,  $\Delta H_s$  and  $\Delta H$  are positive and that  $\Delta H > \Delta H_s$ ; the numerator of (14) is then always positive. Further we distinguish the two cases:

A.  $\Delta V$  is always positive and greater than  $\Delta V_s$ .

B. If we represent the composition of the liquid  $L$ , saturated with solid  $Y$  by:  $y$  Mol  $Y + (1-y)$  Mol. water, then for small values of  $y$   $\Delta V$  is positive and larger than  $\Delta V_s$ ; for greater values of  $y$ , however,  $\Delta V$  is negative. Previously we found (formula 10<sup>a</sup> of Communication XIX) for  $\Delta V$ :

$$\Delta V = \frac{V - yV_y}{1 - y} \dots \dots \dots (15)$$

in which  $V$  and  $V_y$  represent the molecular volumina of the liquid  $L$  and of the solid substance  $Y$ . From this and from the considerations in



Communication XIX it appears that the case *A* can occur only then, when the solid substance *Y* melts with increase of volume, and case *B* when this melting takes place with decrease of volume.

*A.* In the case *A* numerator and denominator of (14) are always positive; consequently the *PT*-curve of the osmotic equilibrium (12) is a curve ascending with the temperature, f.i. curve *aa*<sub>1</sub> of fig. 1. In order

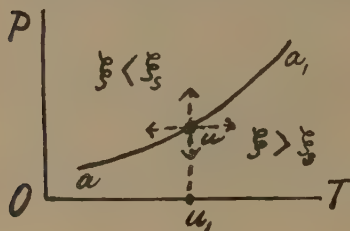


Fig. 1.

to keep the system (12) in osmotic equilibrium with increase of temperature we also have to increase the pressure on both sides of the membrane.

As, therefore, the osmotic equilibrium (12) only exists at temperatures and under pressures, which are represented by points of the *PT*-curve, we may put the question: what will take place, if we choose

a temperature and a pressure, which are represented by a point which is not situated on the *PT*-curve. Let us take the osmotic equilibrium:

$$[Y + L \mid L_s]_{p_u, T_u} \dots \dots \dots (16)$$

which is represented in fig. 1 by the point *u* of the *PT*-curve *aa*<sub>1</sub>. While the pressure remains constant, we bring the temperature to *T*<sub>*u*</sub> + *dT*; we then have the osmotic system:

$$[Y + L \mid L_s]_{p_u, T_u + dT} \dots \dots \dots (17)$$

which is represented in fig. 1 by a point in the vicinity of *u* and on a horizontal line going through this point *u*. We may imagine this point, which we shall call *u'*, in one of the extreme points of the horizontal arrows, drawn through point *u*; for *dT* > 0 *u'* is situated at the right, for *dT* < 0 at the left of the point *u*.

As *dP* is zero in the point *u'*, it follows from (13) for the change of the O.W.A. at the left and the right of the membrane:

$$d\xi = \Delta H \cdot dT \quad d\xi_s = \Delta H_s \cdot dT \dots \dots \dots (18)$$

in which, according to our assumption  $\Delta H > \Delta H_s$ . Consequently, as is indicated also in fig. 1, for *dT* > 0 is:  $\xi > \xi_s$ ; for *dT* < 0 is  $\xi < \xi_s$ .

If we bring, while the temperature remains constant, the pressure to *P*<sub>*u*</sub> + *dP*, then we have the osmotic system:

$$[Y + L \mid L_s]_{p_u + dP, T_u} \dots \dots \dots (19)$$

which is represented in fig. 1 by a point in the vicinity of *u* and in one of the extreme points of the vertical arrows, drawn through point *u*. As now *dT* is zero, follows from (13) for the change of the O.W.A. at the left and at the right of the membrane:

$$d\xi = -\Delta V \cdot dP \quad d\xi_s = -\Delta V_s \cdot dP \dots \dots \dots (20)$$

in which, in accordance with our assumption is  $\Delta V > \Delta V_s$ . For  $dP > 0$  is, therefore,  $\xi < \xi_s$  and for  $dP < 0$  is  $\xi > \xi_s$ .

We may summarise this and the previous in the following way. If in the osmotic system:

$$[Y + L \mid L_s]_{P', T'} \dots \dots \dots (21)$$

$P'$  and  $T'$  are a pressure and a temperature, which are represented:

1. by a point of the  $PT$ -curve, then the system is in osmotic equilibrium and nothing happens;

2. by a point at the right or below the  $PT$ -curve (fig. 1) then is  $\xi > \xi_s$  and consequently water diffuses from right to left;

3. by a point at the left or above the  $PT$ -curve (fig. 1) then is  $\xi < \xi_s$  and consequently water diffuses from left to right.

This diffusion of water, with which each of both the separate systems changes its total composition, continues till the  $O.W.A.$  on both sides of the membrane, is equal again.

B. As, according to our supposition, the denominator of (14) is positive for small values of  $y$ , therefore, at low temperature and pressure, but negative with greater values of  $y$ , this must be zero in a definite point  $b$  of the  $PT$ -curve, consequently at a definite temperature  $T_b$  and under a pressure  $P_b$ . Therefore, the  $PT$ -curve has in the point  $b$  a vertical tangent and consequently it consists of a branch  $a_b$ , ascending with the temperature and of a branch  $a_1 b$  descending with the temperature (fig. 2). On branch  $ab$  is  $\Delta V > \Delta V_s$  and consequently the denominator of (14) is positive; on branch  $a_1 b$  is  $\Delta V < \Delta V_s$  and consequently the denominator of (14) is negative.

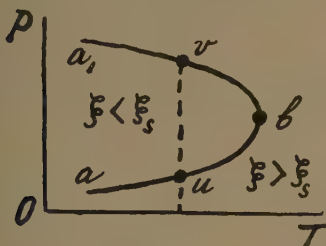


Fig. 2.

Therefore, the osmotic equilibrium (12) can exist no more at temperatures higher than  $T_b$ ; at lower temperatures it can

occur at every temperature under two different pressures.

In a similar way as above we now deduce with the aid of (18) and (20): if in the osmotic system:

$$[Y + L \mid L_s]_{P', T'} \dots \dots \dots (22)$$

$P'$  and  $T'$  are a temperature and pressure, which are represented:

1. by a point of the  $PT$ -curve, then the system is in equilibrium and nothing happens;

2. by a point outside the  $PT$ -curve (fig. 2), then is  $\xi > \xi_s$  and consequently water diffuses from right to left;

3. by a point within the  $PT$ -curve (fig. 2), then is  $\xi < \xi_s$  and consequently water diffuses from left to right.

We are able to deduce the above also in another way, f.i. with the

aid of the  $P\xi$ -diagram, which we have already discussed in Communication XXI. The system  $Y+L$  has viz. at each  $T$  and  $P$  a definite  $\xi$ ; if we change either the  $T$  or the  $P$ , then, therefore, also changes the  $\xi$  viz. the O.W.A. Under constant pressure we are able to represent the connection between the  $T$  and the O.W.A. in a  $T\xi$ -diagram by a curve, which we may call the " $T\xi$ -curve" of the system. If we keep the temperature constant, then we can represent in a  $P\xi$ -diagram the relation between the pressure and the O.W.A.; we then get the  $P\xi$ -curve of the system.

In fig. 1, XXI is represented the  $P\xi$ -curve of a system  $Y+L$  by curve  $cfl$ ; this is deduced in the supposition that the osmotic increase of volume  $\Delta V$  of this system is positive under lower pressures, that it is zero in the point  $f$  and negative under higher pressures. As this supposition is the same as in the case  $B$ , discussed above, the  $P\xi$ -curve in this case  $B$  will have also a form as curve  $cfl$  in fig. 1, XXI; in fig. 4 is drawn also a similar curve  $cfl$ .

If, however, as we have done in case  $A$ , we assume that  $\Delta V$  is always positive, then follows, as  $d\xi = -\Delta V \cdot dP$ , that the  $P\xi$ -curve is a curve, descending with the pressure; as curve  $cul$  in fig. 3.

We now shall indicate in those figs. 3 and 4 also the  $P\xi$ -curve of the liquid  $L_s$ ; as, according to our supposition,  $\Delta V_s$  is positive, it is also a curve, descending with the temperature; in both figures it is represented by the dotted curve  $st$ .

A. In the case mentioned above sub  $A$  fig. 3 is valid, in which  $u$  is the point of intersection of the two  $P\xi$ -curves; of course this figure is valid only for a definite temperature, which we shall call  $T_u$ . The change of the O.W.A. of the system  $Y+L$  and that of the liquid  $L_s$  is defined at constant temperature by:

$$d\xi = -\Delta V \cdot dP \quad d\xi_s = -\Delta V_s \cdot dP \quad . \quad . \quad . \quad (23)$$

in which, according to our supposition  $\Delta V > \Delta V_s$ ; hence follows that on the right side of the point of intersection branch  $ul$  is situated below branch  $ut$  (viz.  $\xi < \xi_s$ ) and on the left side of this point curve  $cu$  is situated above  $su$  (viz.  $\xi > \xi_s$ ); consequently both curves are situated with respect to one another as is drawn in fig. 3 and they have only one point of intersection, therefore. This point, in which is  $\xi = \xi_s$ , represents the pressure

$P_u = Oz$ , belonging to the temperature  $T_u$ , under which the O.W.A. is the same on both sides of the membrane; consequently  $T_u$  and  $P_u$  are corresponding temperature and pressure of the  $PT$ -curve of the osmotic equilibrium (12); they are represented in fig. 1 by point  $u$ .

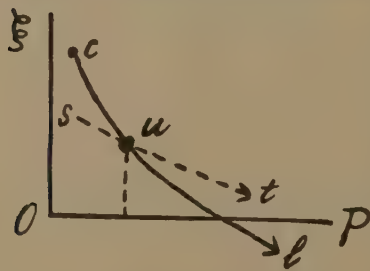


Fig. 3.



If we take in fig. 3 a pressure smaller than  $P_u = Oz$ , then is, as branch  $cu$  is situated above  $su$ ,  $\xi > \xi_s$ ; for pressures greater than  $P_u$  follows  $\xi < \xi_s$ . Going in fig. 1 from point  $u$  towards lower pressures, we must arrive in the field, where is  $\xi > \xi_s$ ; going towards higher pressures, we must arrive in a field, where is  $\xi < \xi_s$ . This is in accordance with the position of both fields, which we have deduced already above in another way.

As we have seen already above, fig. 3 is only valid for a definite temperature, which we called  $T_u$ . If we bring the temperature to  $T_u + dT$ , then both the  $P\xi$ -curves change a little their position and form; from (13) follows for constant pressure:

$$d\xi = \Delta H \cdot dT \quad d\xi_s = \Delta H_s \cdot dT \quad . \quad . \quad . \quad (24)$$

by which the vertical shifting of each point of both the curves is defined. As  $\Delta H$  and  $\Delta H_s$  both are positive, both curves shift towards above, therefore, with increase of temperature; as, according to our supposition  $\Delta H > \Delta H_s$ , each point of curve  $cl$ , therefore, shifts somewhat more towards above than the corresponding point of curve  $st$ ; the point of intersection  $u$  of the two curves is displaced, therefore, a little towards the right, consequently towards a higher pressure. Hence appears, therefore, that in case *A* the pressure of the osmotic system increases with increase of temperature; in accordance with previously we find, therefore, that the  $PT$ -curve consists of a curve ascending with the temperature, as in fig. 1.

*B.* In the case, mentioned sub *B* is valid fig. 4, in which both the  $P\xi$ -curves intersect one another in the two points  $u$  and  $v$ ; just as above we find that both curves are situated with respect to one another as is drawn in fig. 4; the left point of intersection  $u$  is situated always between  $c$  and  $f$ ; the right point of intersection  $v$  can be situated as well between  $u$  and  $f$  as between  $f$  and  $l$ .

As in each of both points of intersection  $\xi = \xi_s$ , therefore at the

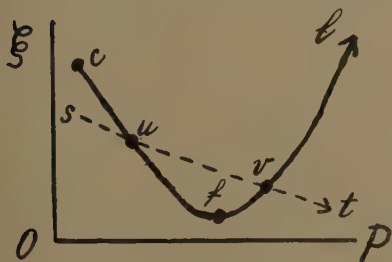


Fig. 4.

temperature, for which fig. 4 is valid, two pressures  $P_u$  and  $P_v$  exist, under which the osmotic equilibrium (12) can exist; this is in accordance also with fig. 2, in which  $u$  and  $v$  represent the corresponding points  $u$  and  $v$  of fig. 4.

If we take in fig. 4 a pressure smaller than  $P_u$  or greater than  $P_v$ , then is, as appears from the position of both curves,  $\xi > \xi_s$ ; if

we take a pressure between  $P_u$  and  $P_v$  then we see that  $\xi < \xi_s$ . Consequently in fig. 2 we must have a field, in which  $\xi > \xi_s$  below point  $u$  and above point  $v$ ; between point  $u$  and point  $v$  must be situated a

field, in which  $\xi < \xi_s$ ; this is in accordance with the position of those fields, which we have deduced above already.

As, with increase of temperature, both curves of fig. 4 shift towards above, but each point of curve  $cfl$  more than the corresponding point of curve  $st$ , the point of intersection  $u$  must be removed towards the right and the point of intersection  $v$  towards the left. With increase of  $T$ , therefore, a temperature comes, at which the two curves touch one another in a point  $b$ , situated between  $c$  and  $f$ . In accordance with fig. 2 we find, therefore, that a definite temperature  $T_b$  exists, at which the osmotic system (12) can exist still only under one single pressure, viz.  $P_b$ .

If we raise the temperature still further, then the two  $P\xi$ -curves do no more intersect one another; consequently the osmotic equilibrium can exist no more. In accordance with fig. 2 also appears that  $\xi > \xi_s$ , so that (12) now is an osmotic system, in which water will diffuse from left to right as long as till a new system is formed, in which the O.W.A. is equal on both sides of the membrane.

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*(To be continued).*

**Serology.** — *Concerning the Transmutant of the Tubercle-bacillus BTTx as Tuberculosis-diagnosticum.* By C. H. H. SPRONCK, K. HOEFNAGEL, W. HAMBURGER and A. J. BOEKELMAN.

(Communicated at the meeting of March 26, 1927).

After the discovery of GRUBER—WIDAL's test for typhus abdominalis researchers tried to perform this serum-reaction also in cases of tuberculosis. In 1898 ARLOING and COURMONT reported at the Congress for Tuberculosis at Paris, that they had succeeded in this effort by employing tubercle-bacilli that grow homogeneously in glycerin-bouillon. The difficulty, however, was that they had to work with low serum-dilutions. They considered the reaction as positive and specific already when agglutination took place in the dilution 1 : 5. Agglutination in the dilution 1 : 20 was a high titre, and the titre 1 : 30 was seldom reached.

Fine emulsions of tubercle bacilli obtained in diverse ways (ROBERT KOCH, V. BEHRING and others) yielded about the same result.

Erelong it appeared that neither the tubercle bacilli of ARLOING and COURMONT, nor artificial emulsions answered the expectation. On the one side the result of the agglutination-test turned out negative in a rather considerable number of indubitable cases of tuberculosis, on the other side positive results were not infrequently obtained in patients with other diseases or in apparently healthy persons. This condemned the method as to its practical use. Before long it was abandoned altogether and replaced by complement-fixation. Most researchers hold that the agglutinating action of human serum on the tubercle-bacillus is not owing to immune-agglutinins but to normal- or para-agglutinins.

Tubercle-bacilli are no doubt agglutinable, for by treating men and animals with killed tubercle-bacilli the blood gets rich in immune-agglutinins, and the agglutination-titre can rise high. But the agglutinability of the tubercle-bacilli is apparently too slight to distinguish small quantities of immune-agglutinins from normal-, and para-agglutinins.

In 1921 there appeared two more papers on agglutination of the tubercle-bacillus. VAUDREMER<sup>1)</sup> had discovered a variation of the tubercle-bacillus, which was agglutinated by a highly diluted serum from tuberculous patients. VAUDREMER's variation consisted of non-acidfast GRAM-positive elements, appearing partly as ramified and moniliform bacilli with swollen ends, partly as ovoid forms and diplococci. The agglutination titre of the serum from tuberculous patients could be as high as 1 : 500. No investigations were made whether this variation could serve practical diagnostic purposes.

<sup>1)</sup> A. VAUDREMER, C.R. de la Société de Biologie, T. 85, p. 1055, 1921.



The second publication is by W. FORNET<sup>1)</sup>, who ungreased tubercle-bacilli with aether-vapour at 40° C. for 6—8 hours, by which process the bacilli lost a great part of their acid-fastness. The sera from 132 tuberculous patients gave in 93 % of the cases positive agglutination in dilutions from 1 : 60 to 1 : 800, whereas the sera from 44 non-tuberculous subjects did not reveal this reaction in 95 % of the cases.

FORNET's diagnosticum, however, seemed to have peculiar defects. According to BIGNAMI<sup>2)</sup> the reaction not seldom turned out positive with apparently healthy persons and was almost invariably positive with luetici. Furthermore the reaction was not infrequently negative in tuberculous patients. Strange to say, BIGNAMI could not find any rods in the emulsion. In a second paper<sup>3)</sup> this observer recorded that FORNET's diagnosticum was made up of a 6 % solution of monopotassium-phosphate and a small quantity of acid-fast rods. KOHLER<sup>4)</sup> found the agglutination-titre in patients suffering from malignant tumours sometimes higher, then again low. According to SZYMENSKI<sup>5)</sup> the emulsion was as clear as water! SALUS<sup>6)</sup> found a high titre in the puerperium and also in gravidae. We may conclude therefore, that FORNET's diagnosticum has proved to be impracticable for unknown reasons.

New findings are not infrequently due to chance. Truth to tell, this also holds good for our tuberculosis-diagnosticum. By growing tubercle bacilli on media, whose composition was gradually altered, SPRONCK and HAMBURGER<sup>7)</sup> obtained from three typical strains (1 bovine and 2 human) the same variation with qualities not less stable than those of the original strains. This induced them to consider the altered bacillus as a true transmutant, which was called tuberculosis transmutatus x (BTTx). It was naturally surmised that an error, contamination, symbiosis etc. had come into play. In order to remove every doubt they tried to find new evidence. The serum from tuberculous patients was examined for agglutinins. But they met with a difficulty, viz. that one serum had a high titre, the other a low one. To elucidate this fact a systematic investigation of many sera was necessary. The result of this investigation was a perfect success. The transmutant proved to be a suitable tuberculosis diagnosticum, and this is at the same time a new and reliable proof of its true descent from the tubercle bacillus.

We have worked with young transmuted tubercle bacilli, 24—48 hours old, grown on a solid agar-medium and washed with physiological salt-

1) W. FORNET, Annales de l'Inst. PASTEUR, T. 35, p. 797, 1921 and Deutsches Arch. f. Klin. Med. Bd. 138, p. 229, 1922.

2) G. BIGNAMI, Tuberculosi Bd. 14, N<sup>o</sup>. 12, 1922.

3) G. BIGNAMI, Tuberculosi Bd. 15, N<sup>o</sup>. 4, 1923.

4) A. KOHLER, Berl. Klin. W. Jg. 2, N<sup>o</sup>. 14, 1923.

5) N. SZYMENSKI, Schw. med. W., Jg. 54, 1924.

6) G. SALUS, Med. Klinik, Jg. 20, N<sup>o</sup>. 10, 1924.

7) C. H. H. SPRONCK and W. HAMRURGER. Geneeskundige Bladen, Reeks 25, N<sup>o</sup>. 3, 1926.

solution, to which 0.2 % formol had been added. In this way it is easy to get homogeneous suspensions, which show no trace of spontaneous agglutination and subside slowly. Even through brisk centrifugation it is not easy to clarify the suspensions. The agglutinability remains unchanged for weeks, but we have been careful to use only suspensions, not older than 14 days. Living and dead bacilli are equally agglutinable. In the formol-suspensions the bacilli are killed off only after a few days, more rapidly in the incubator than at room-temperature. Furthermore we took care that the denseness of the suspensions was always equal. To prevent errors incident to inhibition, the sera were allowed to stand for some days. If one wishes to examine fresh serum, heating for  $\frac{1}{2}$  hour at 55° C. is necessary. We used exclusively the following six serum-dilutions: 1:10, 1:50, 1:100, 1:200, 1:300, and 1:400. We did not go further in determining the titre-limit. We did not like to enter into details, because our chief concern was to ascertain whether in applying the transmutant the stumbling-block of the irregularities should disappear, that induced us to abandon the earlier methods. Details would be in place, as we thought, only after a solid basis had been established on which to build.

Of each dilution 2 cc. were put into ordinary agglutination-tubes, to each tube 0.1 cc. suspension was added and after stirring with a glass rod the tubes remained standing at 38° C. After 3 hours readings were taken with the hand-lens. After the tubes had stood at room-temperature till the next day, readings were taken again. As a rule the reaction is accomplished within 2 hours with coarse flocculation. The clumps can subside rapidly so that already after 2 hours a white deposit lies on the bottom and the supernatant fluid is cleared. But the reaction may be tardy, when finer clumps are formed that sink more slowly, and in this case the results of the two readings may be different. If for instance, after standing for 3 hours at 37°, it is still doubtful whether the titre 1:200 has been reached, a precipitation may be distinctly visible in this dilution the next day.

The distinctness of the agglutination is equal to that with GRUBER—WIDAL's test. No precipitation is noticeable after 24 hours either in the control-tubes or in the tubes in which agglutination does not occur. Even after 2, nay 3 days the result can still be read.

The 4 transmuted strains at our disposition when starting the investigation (their present number amounts to 7) appeared not to be equally agglutinable. The human strains VI and VIII gave most satisfactory results. We have used strain VI because its growth was quicker and more luxuriant than that of VIII. At present this difference does not exist any more.

We examined sera from men, cows and horses, By far most human sera were taken from adults, partly our own patients, attended at St. Antonius-Hospital at Utrecht. We also received sera from our

colleagues J. H. AKKERMAN, A. J. G. ATHMER, S. G. T. BENDIEN, G. J. DE BOER, G. J. BON, W. F. ENKLAAR, H. B. GOETSSCH, J. HOOGKAMER, L. C. KERSBERGEN, E. S. M. VAN LIER, J. M. ROELVINK, P. ROORDA, R. RUTTEN, and B. SUERMONDT, to all of whom we feel indebted for their kind assistance.

Table I illustrates the indubitable cases of tuberculosis; table II the

TABLE I. Tuberculous-patients.

Nº.	Diagnosis not doubtful.	Titre	
1	Pulmonary Tuberculosis far-advanced	<1:10	
2	" " "	<1:10	
3	" " "	1:10	
4	" " "	1:10	
5	" " "	1:10	
6	Surgical Tuberc. far-advanced	1:10	
7	" " "	1:50	
8	Pulmon. Tuberc. far-advanced	1:50	
9	" " "	1:50	
10	Old lung-process stationary	1:50	
11	Pulmon-Tuberculosis	1:100	
12	" "	1:100	
13	" "	1:100	
14	" "	1:100	
15	" "	1:100	
16	" "	1:100	
17	" "	1:100	
18	Surgical Tuberculosis	1:100	
19	" "	1:100	
20	" "	1:100	
21	" "	1:100	
22	Incipient Pulmon-Tuberculosis	1:200	
23	Pulmonary Tuberculosis	1:200	
24	Peritonitis tuberculosa	1:200	
25	" "	1:200	
26	" "	1:200	



TABLE II. Tuberculous-patients?

Nº.	Diagnosis	Titre	Notes
1	Peritonitis tuberculosa ?	1 : 10	
2	" " ?	1 : 10	v. Pirquet —
3	Tuberculous Otitis ?	1 : 10	" +
4	Vague complaints, tuberculosis ?	1 : 10	
5	Tuberculosis ? Difference of opinion between two physicians	1 : 10	
6	Adnexitis tuberculosa aut gonococcica ?	1 : 10	
7	Suspicious swelling of the cervical glands	1 : 10	
8	Tuberculosis ? Difference of opinion between two physicians	1 : 10	
9	Struma. Plus tuberculosis ?	1 : 10	
10	Tuberculous bronchial glands ?	1 : 10	
11	Vague complaints	1 : 10	
12	Incipient pulmonary tuberculosis ?	1 : 50	
13	Peritonitis tuberculosa ?	1 : 50	
14	" " ?	1 : 50	
15	Tuberculous ear-suppuratation ?	1 : 50	
16	Peritonitis tuberculosa ?	1 : 50	
17	Pulmonary-tuberculosis ?	1 : 100	v. Pirquet +
18	" " ?	1 : 100	" +
19	" " ?	1 : 100	" +
20	Morbus Addisonii. Tuberculosis ?	1 : 100	
21	Suspicious noduli near the lung-hilus	1 : 100	
22	Morbus Basedowi. Also tuberculosis ?	1 : 100	
23	Abdominal-tuberculose ?	1 : 100	v. Pirquet +
24	Suspicious ear-suppuratation ?	1 : 100	" +
25	Tuberculous ear-suppuratation ?	1 : 100	" +
26	Pulmonary-tuberculosis ?	1 : 100	
27	Vague complaints. Tuberculosis ?	1 : 100	
28	Suspicious symptoms	1 : 100	
29	Pulmonary tuberculosis ?	1 : 200	v. Pirquet weak +
30	Incipient pulmonary-tuberculosis ?	1 : 200	" +
31	Habitus phthisicus. Coughing	1 : 200	
32	Pulmonary hemorrhage or simulation ?	1 : 300	v. Pirquet —
33	Suspicious chronic bronchitis	1 : 400	no bacilli found in sputum

TABLE III. Sufferers from divers diseases or apparently healthy men.

Nº.	Diagnosis	Titre	
1	Nephrolithiasis	<1:10	
2	Carcinoma ventriculi	<1:10	
3	Nephritis	<1:10	
4	Hypertrophia prostatae	<1:10	
5	Myelogenic leukaemia	<1:10	
6	Paroxysmal hemoglobinuria	<1:10	
7	Carcinoma ventriculi	<1:10	
8	Rheumatismus chronicus	1:10	
9	Apparently healthy	1:10	
10	Lues	1:10	
11	Nephritis	1:10	
12	Biliary colic	1:10	
13	Carcinoma ventriculi	1:10	
14	Streptococcal sepsis	1:10	
15	Taboparalysis	1:10	
16	Carcinoma recti	1:10	
17	Gravida. Apparently healthy	1:10	
18	Carcinome of the port. vag. uteri	1:10	
19	Fibromyoma uteri	1:10	
20	Pneumonia and erysipelas	1:10	
21	Struma	1:10	
22	"	1:10	
23	Chronic rheumatism	1:50	
24	Congenital lues	1:50	
25	Hypertension	1:50	
26	Hernia inguinalis	1:50	
27	Headaches	1:50	
28	Apparently healthy	1:50	
29	Chronic rheumatism	1:100	
30	Achylia gastrica	1:100	
31	Morbus Basedowi	1:100	
32	Fibromyoma uteri	1:100	
33	Chronic rheumatism	1:100	
34	Ascarides	1:100	
35	"	1:300	

suspected cases, while table IV gives a survey of patients with divers diseases or apparently healthy subjects. The cases of each group are arranged according to the titre found, so that it is easier to survey the results, and (which is the chief thing) the difference between the results of the three groups can be seen immediately.

With 26 tuberculous patients (Table I) the titre was in 16 cases (61.5 %) 1:100 to 1:200; in 4 cases 1:50; in 4 cases 1:10; and in 2 cases smaller than 1:10.

With 35 persons, suspected to be suffering from tuberculosis, (Table II) the titre appeared to be in 17 cases (48.4 %) 1:100 to 1:400; in 5 cases 1:50; and in 11 cases 1:10.

With 35 men suffering from other diseases or apparently healthy (Table III) the titre amounted in 7 cases (20 %) to 1:100 or 1:300; in 6 cases 1:50; in 16 cases 1:10 and in 6 cases it was smaller than 1:10.

A comparison of the three tables tells us that the titres 1:100 and higher are in the majority in Table I, in Table III in the minority and that in Table II they are nearly as numerous as the lower titres. On the other hand the number of titres 1:50 is about the same in the three tables.

In Table I (the group of the tuberculous patients) the great difference between the titres is remarkable, ranging from lower than 1:10 to 1:200. In far-advanced cases as well as in stationary or convalescent cases the titre appears to be lower than 1:100, in all other cases it is 1:100 or higher.

That the titre goes down in far-advanced cases had already been stated by ARLOING and COURMONT. The agglutinins disappeared from the blood, while also the tuberculin-reaction then became negative. This may be caused either by decreased production of antibodies resulting from exhaustion or increased consumption. It may be imagined that the agglutination-titre goes down when the tuberculous process is stationary or when the patient is convalescent.

In the group of suspicious cases (Table II) the titre is 1:100 or higher in nearly half the cases. In case N<sup>o</sup>. 32, with a titre of 1:300, the symptoms were so vague that we doubted whether hemoptysis was feigned or not. The patient with a titre of 1:400 suffered from a somewhat doubtful attack of bronchitis; no tubercle-bacilli were found in the sputum.

In case of an agglutination-titre 1:100 or higher the tuberculin-reaction appeared, as a rule, to be positive. Exceptionally, however, it was only slightly positive or even negative. It is remarkable that this exception occurred only in cases with a titre 1:200 (N<sup>o</sup>. 29) and 1:300 (N<sup>o</sup>. 32). If this exception should really appear to be due to a high titre, the research after the property of the serum of these patients to neutralize

the tuberculin, may give important results for our knowledge about the antibody of tuberculin (antituberculan).

In the group of persons suffering from divers other diseases or apparently healthy men (table III) the large number of cases with a titre of 1 : 10, or still lower (22 = 66.6 %), is remarkable. But it is not less remarkable that in this table 7 cases occur with high titres, this looks rather serious.

Of course, we did our utmost, and are doing so still, to ascertain whether in these cases a tuberculous process really exists.

An inquiry into the case of achylia gastrica (N<sup>o</sup>. 30) yielded an astonishing result. The patient (a woman) had just spat some blood for the first time. After this she began to cough and expectorate, and a cavia-experiment proved the presence of tubercle-bacilli in the sputum.

In the case of morbus Basedowi (N<sup>o</sup>. 31) the tuberculin-reaction appeared to be markedly positive and röntgenoscopy revealed a focus in the lungs of a positively tuberculous character. In fact it is notorious that an increased secretion of the thyroid gland is often complicated with a tuberculous pulmonary process of a comparatively benign character. A paper was published lately from the clinic of Prof. VON EISELSBERG at Vienna, on the frequent occurrence of this complication<sup>1)</sup>.

A röntgen-examination also revealed in two patients with ascarides (N<sup>o</sup>. 34 and 35) deviations of the lungs that bespeak tuberculosis.

The case of chronic rheumatism N<sup>o</sup>. 29 concerns a girl of 19 years with pains in the extremities and habitus phthisicus. She does not cough. The attending physician assumes the possibility of tuberculous rheumatism, because the pains disappear in the intervals of rest.

We are still intent on examining the three lastnamed cases and the the three remaining ones of this group. Our results of further research and observation also relating to the patients of table II with a titre of 1 : 100 and higher, will be published in due time.

Before entering upon our systematic investigation we had already examined a number of human sera to orientate ourselves. These cases are not comprised in the tables, first because we had worked with other dilutions and secondly because the sera were partly taken from patients, who had already been treated with transmuted tubercle-bacilli, which causes a rise of the titre.

On the basis of the observations reported here we conclude, that the agglutination-experiment with the transmutant is a suitable test for the recognition of a tuberculous process in man. The results achieved in the groups II and III favour the view, that the agglutination titre is high already in the first stage of the disease, so that there is some prospect that by means of the transmutant the disease can be diagnosed betimes.

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<sup>1)</sup> O. SÜSANI, Mitteilungen aus den Grenzgebieten der Medizin und Chirurgie, Bd. II. H. 2, 1927.



Provisionally we look upon the titre 1:100 as the lowest titre-limit pointing to tuberculosis. We say provisionally, as it is necessary to determine this limit more precisely. Since there is a basis solid enough to make it worth while continuing our investigations, we proceed to ascertain in the first place the significance of titres between 1:50 and 1:100. If such a titre is found in a suspicious case, it is desirable to repeat the inquiry after some time. When it is found that the titre has risen, this would then indicate tuberculosis and progression even though the titre 1:100 should not yet be reached.

From the result of the examination of sera from cows and horses it appears that the agglutinability of the transmutant is about 20 times greater than that of the acidfast tubercle bacilli. So if we consider agglutination through human serum in the dilution 1:100 to be positive and specific, this agrees quite well with the titre-limit 1:5 indicated by ARLOING and COURMONT. They considered 1:20 as a high titre and the 20 times higher titre (1:400) is the highest we found thus far. As ARLOING and COURMONT still exceptionally met with higher titres than 1:20 we suppose that the transmutant will also show titres of 1:500 and 1:600.

It seems doubtful whether the difference between the agglutinability of the acidfast tubercle-bacillus and that of the transmutant is only a quantitative one. Perhaps an examination of the serum from patients with typhus or pneumonia will tell us something about this, because according to BESANÇON and SARBONNES<sup>1)</sup> in such cases ARLOING and COURMONT's homogeneous tubercle-bacilli were not infrequently agglutinated in very strong serum-dilutions.

The marked agglutinability of the transmutant must be owing to the fact that the adsorption of the immune-agglutinins is promoted considerably by the absence of the wax-cover enveloping the typical tubercle-bacillus. The transmuted tubercle-bacilli contain so little fat or waxy substance that they burn away in the flame without any crackling. This conception is supported by the fact that also the above-mentioned non-acidfast GRAM-positive variation of the tubercle bacillus of VAUDREMER<sup>2)</sup> appeared to be particularly agglutinable, and that FORNET<sup>3)</sup> could render typical tubercle-bacilli very agglutinable by treatment with aether-vapour. That the ectoplasm can impede the agglutination is also shown in FRIEDLÄNDER's bacillus, whose inagglutinability disappears with a decrease of the size of the mucous cover under the influence of subcultivation.

The wax-cover of the tubercle-bacillus is, in our opinion not only answerable for its small agglutinability and its uselessness for complement-

<sup>1)</sup> Compt. rend. de la Société de Biologie, T. 67, p. 548, 1909.

<sup>2)</sup> L.c.

<sup>3)</sup> L.c.

fixation, but it also stunts its growth and hinders its application in vaccinothrapy.

According to the researches by BURGI<sup>1)</sup> and MAMLOK<sup>2)</sup> the serum from the cow is richest in normal-agglutinins, which agglutinate all sorts of bacteria in a high degree. Next to the cow come the horse, the pig, the sheep, the fowl, the goose, the pigeon, the dog, the rabbit, man., the rat and the cavia.

So it could be anticipated that the agglutination-reaction with the transmutant would prove unfit to distinguish the healthy from the tuberculous cow. But ARLOING and COURMONT had recorded that by means of their homogeneous tubercle-bacilli tuberculous cows could easily be distinguished, which, however, was denied by BECK and RABINOWITSCH<sup>3)</sup>. As it was possible that this divergence of opinions might result from using different homogeneous strains with different agglutinability, we have also examined the agglutinating action of the serum from healthy and from tuberculous animals on the human transmutant VI.

In most cases the animals were bled before being killed. Whether the cow was tuberculous or not, appeared at the post-mortem examination. Furthermore we also examined the serum from two cows suffering from tuberculosis, which were in Prof. WESTER's clinic at the Utrecht university. It is a pleasure to mention our indebtedness to Prof. WESTER, and Dr. BEIJERS, and to Mr. RUTGERS and Dr. DE GRAAF of the Utrecht abattoir for their kind collaboration.

Table IV gives a general view of the agglutination-titres in tuberculous and non-tuberculous cows. It will be seen, that the titres range from 1 : 100 to 1 : 600. In other words the serum from the cow contains such a large quantity of normal-agglutinins that the transmutant is useless. By using ARLOING and COURMONT's homogeneous tubercle-bacilli BECK and RABINOWITSCH achieved the same result in 1901. The sera from healthy and tuberculous cows had the same titres. From their protocols it appears that the lowest titre was 1 : 5, the highest 1 : 30. The lowest titre found by us being 1 : 100 and the highest 1 : 600 it is evident, that the agglutinability of the transmutant is 20 times that of acidfast tubercle-bacilli.

According to LANDSTEINER and others the normal-agglutinins disappear by the immunisation process and are replaced by immune-agglutinins. Since it has repeatedly been asserted that the former are less resistant to heating than immune-agglutinins, we have availed ourselves of this opportunity to ascertain the influence of heating upon the titres of the sera from tuberculous, and non-tuberculous cows. The result is, that by

1) BÜRGI, Archiv f. Hygiene, Bd. 62, p. 239.

2) MAMLOK, Archiv f. Hygiene, Bd. 68, p. 95.

3) M. BECK und L. RABINOWITSCH, Deutsche med. Wochenschr. 1901, N<sup>o</sup>. 10, S. 145.

TABLE IV. Tuberculous and non-tuberculous cows.

Protocol No.	Diagnosis	Titre
1	Inspection: tuberculosis of a retropharyngeal lymphatic glands	1:400
2	Inspection: three tuberculous lymphatic glands (two retropharyngeal and one bronchial)	1:600
3	Clinical diagnosis: chronic tympanitis in consequence tuberculous, mediastinal lymphatic glands. The section verified the diagnosis	1:100
4	Inspection: tuberculous, mediastinal lymphatic glands	1:400
5	Inspection: no tuberculosis	1:400
6	Inspection: tuberculous bronchial and mesenterial lymphatic glands	1:400
7	Inspection: no tuberculosis	1:600
8	Inspection: tuberculous retropharyngeal lymphatic glands, one of which softened	1:600
9	Inspection: no tuberculosis	1:600
10	Inspection: no tuberculosis	1:400
11	Inspection: tuberculous bronchial and mediastinal lymphatic glands	1:400
12	Inspection: free from tuberculosis	1:400
13	Inspection: tuberculous mesenterial and hepatic lymphatic glands	1:600
14	Inspection: free from tuberculosis	1:400
15	" " " "	1:600
16	" " " "	1:200
17	" " " "	1:200
18	Clinical diagnosis: grave tuberculosis. Section: tuberculosis of mesenterial and hepatic lymphatic glands, pleura and lungs	1:200

heating the sera, diluted with the physiological salt-solution 1:10, for one hour resp. at 58, 59, and 60° C., the agglutination-titre not only of the sera from the non-tuberculous, but also those from the tuberculous cows, fell and that in approximately the same degree.

Our investigation of the sera from three healthy horses yielded the titres 1:400, 1:600, and 1:1000. We had, moreover the rare opportunity to examine the serum from a tuberculous horse in Prof. WESTER's University Clinic. Afterwards the clinical diagnosis was confirmed by the section. The titre of the serum from this horse was 1:800.

Horse-serum, then, seems to be richer in normal agglutinins than the serum from cows, so that the agglutination-test is useless with horses either. In fact it had long been known that serum from horses, which animals seldom contract tuberculosis, agglutinates acidfast tubercle-bacilli in higher dilution even than cow's serum. ROBERT KOCH e.g. had found in 10 horses 8 times a titre of 1:25 and twice one of 1:50. When comparing these titres with ours, we see here again, that, just as with horses, the agglutinability of the transmutant is about 20 times greater than that of acidproof tubercle-bacilli.

In conclusion we wish to answer the question whether the new diagnosticum can be applied in medical practice. We do not hesitate in answering it in the affirmative. In view of our experience the agglutination-experiment is a simple and useful method to recognize tuberculosis in its early stage, when the complaints are still vague and the symptoms are not only inconclusive for tuberculosis but even remind us hardly or not at all of tuberculosis. If the reaction turns out positive, it can only benefit the patient, for the positive reaction will induce the attending physician to observe the serologic diagnosis closely which may lead to the discovery of a latent tuberculous focus.

While the positive tuberculin reaction indicates infection with tubercle-bacilli, the positive result of the agglutination test is to be considered as an indication of a progressive tuberculous process either existing or having existed some time ago. We do not know how long the production of specific agglutinins continues after a tuberculous process has become stationary or has been cured. Experience will have to teach us this.

It is evident, therefore, that the agglutination-experiment does not enable us to say whether a tuberculous process is "active" or "inactive". Many researchers hold that the problem can be solved by complement-fixation. But this reaction as well only indicates whether and how many antibodies are circulating in the blood, and in our opinion the agglutination-test is more reliable generally speaking, than complement-fixation.

The agglutination-test is not only valuable for diagnostics, it may also be useful when the existence of tuberculosis is unquestionable. For just as complement-fixation, it is a suitable method to determine the patients resisting power anyhow if the tuberculosis process has not been stationary for a considerable time and the patient has not been treated with a specific preparation containing agglutinogens of the tubercle bacillus, as e.g. KOCH's bacillus-emulsion. This treatment, of course, may induce a one-sided increase of the agglutination-titre, and in this case the titre is not in any way indicative of other antibodies, that are of significance for the defense. Agglutinins, as such, are of no importance for the defense but experience teaches us that during immunization the various functions of the antibodies generally increase in the same degree. So if many



agglutinins occur in the serum of a tuberculous patient, the anti-infectious, and anti-toxic action of the blood will, to all appearance, be equal.

Conversely, a fall of the agglutination-titre in a patient with a progressively tuberculous process is a critical symptom, which is regularly observed in far-advanced cases, and may result from a smaller production of antibodies as well as from profuse consumption e.g. through binding with antigens.

As stated before, fall of the titre may also result from a prolonged stationary tuberculous process, or from a curing case, so that no or hardly any agglutinins pass from the tuberculous focus into the blood, and consequently the secretion of agglutinins gradually lessens. The symptoms manifested by the patient preclude a confusion of the causes of the fall alluded to.

Furthermore, the titre-determination can supply important indications for the vaccinotherapeutic application of transmuted tubercle-bacilli. If the titre has already fallen much, little benefit can be expected from vaccinotherapy. Moreover a low titre warns us to be cautious. The fever of tuberculous patients is not chiefly elicited by tuberculin, but by a toxin, which is also present in the transmuted tubercle-bacilli. If the agglutination-titre of a patient is high, his blood possesses also antitoxic properties and he tolerates the injections of comparatively large quantities of transmuted tubercle-bacilli better than a healthy, non-tuberculous individual. If however the agglutination-titre of the sufferer has sunk very low, the injection of a comparatively small quantity of transmuted tubercle-bacilli would evoke fever and other symptoms of poisoning, because the blood does not neutralize the poison of the bacilli.

## SUMMARY.

10. The agglutinability of the transmuted tubercle-bacillus x (BTTx) is about 20 times that of the typical acidfast tubercle-bacillus, which is ascribed to absence of the wax-cover, which impedes the adsorption of the specific agglutinins.

20. The blood-serum of tuberculous patients agglutinates the transmutant in dilutions from 1:100 to 1:400. The titre 1:100 is provisionally considered to be the lowest titre-limit of the positive reaction.

30. In a case of advanced tuberculosis, the agglutination-titre falls and may even become less than 1:10.

40. The titre can sink lower than 1:100 if the tuberculous process has been stationary for a long time already or is recovering.

50. In incipient tuberculosis the agglutination-titre is soon rather high, so that the disease may be recognized through the experiment at a time when there are still few complaints which hardly suggest the existence of tuberculosis.

6°. The serum from sufferers from other diseases, or apparently healthy persons has a titre mostly lower than 1 : 50 or even 1 : 10. The titre 1 : 10 warns us to look out for a tuberculous focus.

7°. In cases though only slightly suspicious the titre 1 : 50 necessitates a repetition of the agglutination-experiment after some time.

8°. If the agglutination-test turns out positive, the tuberculin action of v. Pirquet will also be positive. Exceptionally a weak or negative tuberculin reaction occurs with a positive agglutination.

9°. The sera from cows and horses are so rich in normal agglutinins that the transmutant is not fit to demonstrate the existence of tuberculosis in these animals.

10°. The suitability of the transmutant as tuberculosis-diagnosticum is a new evidence for its real descent from the tubercle-bacillus.

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**Mathematics.** — *The Quadratic Birational Correspondence between Two Linear Complexes of Rays.* By Dr. G. SCHAAKE. (Communicated by Prof. HENDRIK DE VRIES).

(Communicated at the meeting of April 30, 1927).

§ 1. For a quadratic birational correspondence between two linear complexes of rays  $C$  and  $C'$ , the lines  $l'$  that are associated to the lines  $l$  of a plane pencil  $w$  of  $C$  form a quadratic scroll  $\varrho'^2$  of  $C'$ , and, inversely, in such a correspondence a scroll  $\varrho^2$  of  $C$  corresponds to a plane pencil  $w'$  of  $C'$ .

In order to show that such correspondences between two given linear complexes of rays  $C$  and  $C'$  exist and to examine these correspondences, we make use of the representation of NOETHER—KLEIN<sup>1)</sup> of the rays  $l$  of a linear complex  $C$  on the points  $L$  of a three-dimensional space  $R_3$ .

This representation has the following properties. Among the lines  $l$  of  $C$  there is one cardinal line  $l_1$ . The image points of  $l_1$  form a plane  $\alpha$ .  $\alpha$  contains a conic  $k^2$  consisting of singular image points. To any point of  $k^2$  there corresponds a plane pencil of  $C$  which contains  $l_1$ . A scroll in  $C$  of the order  $\nu$  that has a  $\nu$ -fold line in  $l_1$ , is represented on a curve of the order  $\nu-\nu$  that cuts  $k^2$  in  $\nu-2\nu$  points. A curve of the order  $n$  which cuts  $k^2$  in  $s$  points, is the image of a scroll in  $C$  of the order  $2n-s$  that has an  $(n-s)$ -fold line in  $l_1$ . To a congruence  $(\mu, \mu)$  that has a  $\varrho$ -fold line in  $l_1$ , there corresponds a surface of the degree  $2\mu-\varrho$  of which  $k^2$  is a  $(\mu-\varrho)$ -fold conic. To a surface of the degree  $m$  that contains  $k^2$   $m_1$ -fold a congruence of rays  $(m-m_1, m-m_1)$  is associated that has an  $(m-2m_1)$ -fold line in  $l_1$ .

§ 2. First of all we shall prove that between any two given linear complexes  $C$  and  $C'$  a quadratic one-one correspondence may be indicated that associates a given line  $l_1$  of  $C$  to any likewise given line  $l'_1$  of  $C'$ .

With a view to this we suppose the complexes  $C$  and  $C'$  to be represented in the way indicated in § 1 resp. on the three-dimensional spaces  $R_3$  and  $R'_3$  in such a way that  $l_1$  and  $l'_1$  are the cardinal lines of these representations. In this case the planes  $\alpha$  and  $\alpha'$  correspond resp. to  $l_1$  and  $l'_1$ . The lines of  $C$  cutting  $l_1$  are represented on the points of the conic  $k^2$  in  $\alpha$ , the lines of  $C'$  cutting  $l'_1$  on the points of the conic  $k'^2$  in  $\alpha'$ .

Suppose that we have found a correspondence between the lines  $l$  of

<sup>1)</sup> Cf. e.g. STURM, *Liniengeometrie*, 1, p. 261.

$C$  and the lines  $l'$  of  $C'$  which satisfies the above mentioned conditions. If to the image point  $L$  of the line  $l$  of  $C$  we always associate the image point  $L'$  of the line  $l'$  of  $C'$  corresponding to the said line  $l$ , there arises a birational correspondence between the points of  $R_3$  and  $R'_3$ .

Let us determine the order of this correspondence. The points  $L$  of a plane  $\varphi$  of  $R_3$  are the image points of the lines of a bilinear congruence  $\Phi$  of  $C$  which contains  $l_1$ . This has two lines in common with the quadratic scroll of  $C$  that corresponds to a plane pencil of  $C'$ . The congruence of  $C'$  associated to  $\Phi$  contains, therefore, two lines which belong at the same time to an arbitrary plane pencil of this complex. Accordingly to the congruence  $\Phi$  there corresponds in  $C'$  a congruence (2,2) that contains  $l'_1$  and is represented on a cubic surface  $\varphi'^3$  which contains  $k'^2$  as a single conic. Inversely to a plane  $\varphi'$  of  $R'_3$  there corresponds a cubic surface  $\varphi^3$  of  $R_3$  containing  $k^2$ .

The transformation  $(L, L')$  is, therefore, cubical. To the singular points in  $R_3$  belong the points of  $k^2$ , to the singular points in  $R'_3$  those of  $k'^2$ .

The latter also appears thus. A point  $L$  of  $k^2$  is the image point of the lines of a plane pencil of  $C$  containing  $l_1$ . This is transformed into a scroll of  $C'$  containing  $l_2^3$  which is represented on a line of  $R'_3$ . In this way it is also evident that to a point of  $k'^2$  there corresponds a line of  $R_3$ .

If  $L$  describes a line  $r$ ,  $l$  describes a scroll of  $C$  containing  $l_1$ . This has four lines in common with the congruence (2,2) corresponding to a bilinear congruence of  $C'$  and is, therefore, transformed into a biquadratic scroll of  $C'$  that contains  $l'_1$ . Consequently to a line  $r$  of points  $L$  in  $R_3$  a cubic  $r'^3$  of  $R'_3$  is associated which cuts  $k^2$  twice.

§ 3. The general cubic point correspondence between the spaces  $R_3$  and  $R'_3$  is established by the aid of a twisted sextic  $k^6$  of the genus three in  $R_3$ . This curve is the base curve of a linear complex  $\Gamma$  of cubic surfaces. Three surfaces of  $\Gamma$  that do not belong to the same pencil, have one point outside  $k^6$  in common.

We shall now assume a projective correspondence between the cubic surfaces  $\varphi^3$  of  $\Gamma$  and the planes  $\varphi'$  of  $R'_3$ . A point  $P$  of  $R_3$  defines a net of surfaces  $\varphi^3$ . To this a sheaf of planes of  $R'_3$  is associated with vertex  $P'$ , which point we shall associate to  $P$ . Inversely to a point  $P'$  corresponds one point  $P$ . For  $P'$  is the vertex of a sheaf of planes; to this corresponds a net of  $\Gamma$ . The surfaces of this net have one point  $P$  outside  $k^6$  in common, which point corresponds to  $P'$ .

The locus of the singular points in  $R_3$  of the transformation is the curve  $k^6$ . The singular points in  $R'_3$  also form a sextic  $k'^6$  of the genus three. To a point of  $k^6$  or  $k'^6$  corresponds resp. a trisecant of  $k'^6$  or  $k^6$ . The trisecants of  $k^6$  and  $k'^6$  from resp. the scrolls of the eighth degree  $\omega^8$  and  $\omega'^8$ .



§ 4. In order to arrive at the transformation  $(L, L')$  indicated in § 2, we suppose  $k^6$  to be degenerate in  $k^2$  and a biquadratic curve  $k^4$ . We choose a surface  $\varphi^3$  through  $k^2$  and  $k^4$  which through the transformation  $(L, L')$  passes into a plane  $\varphi'$  of  $R'_3$ .

This transformation gives us a representation of  $\varphi^3$  on  $\varphi'$  which has six singular points in  $\varphi'$ , to wit the points of intersection of  $\varphi'$  with the curve  $k'^6$  in  $R'_3$ . In the general cubic transformation  $k^6$  is represented on a curve  $c'^8$ , the intersection of  $\omega'^8$  with  $\varphi'$ , which has the six singular points as triple points. As according to § 2 a curve  $r^3$  of  $\varphi^3$  associated to a line  $r'$  of  $\varphi'$  cuts the conic  $k^2$  twice, the image curve of  $k^2$  in  $\varphi'$  must cut a line  $r'$  of  $\varphi'$  twice; it is, therefore, a conic of  $\varphi'$ . This conic  $c'^2$  passes through four singular points of  $\varphi'$  because for  $c'^2$  four lines must split off from the sextic associated to an arbitrary conic.

Consequently the curve  $k^4$  is represented on a sextic  $c'^6$  which has triple points in two of the singular points and double points in the other four. Hence  $k^4$  must be a rational curve which cuts  $k^2$  in four points as  $c'^6$  and  $c'^2$  have four points in common that are not singular for the representation.

Now a conic  $k^2$  and a rational curve  $k^4$  that cuts  $k^2$  twice, form the base curve of a complex  $\Gamma$  of cubic surfaces  $\varphi^3$  of which three that do not belong to the same pencil, have one point in common outside  $k^2$  and  $k^4$ . We assume, therefore, a collinear correspondence between the surfaces  $\varphi^3$  of  $\Gamma$  and the planes  $\varphi'$  of  $R'_3$  and we consider the cubic transformation that arises if to any point  $P$  of  $R_3$ , which is the base point of a net of surfaces  $\varphi^3$ , we associate the vertex of the sheaf of planes corresponding to it in  $R'_3$ .

The surface  $\omega^8$  of the trisecants of  $k^6$  consists of the surface  $\omega^2$  of the trisecants of  $k^4$  and the surface  $\omega^6$  of the bisecants of  $k^4$  that cut  $k^2$ . The latter surface has a triple curve in  $k^2$  and a double curve in  $k^4$ .

We find the order of the curve of singular points associated to  $\omega^2$  that lies in  $R'_3$ , by determining the number of points of intersection with a plane  $\varphi'$  of  $R'_3$ . These points of intersection are to be found among the six singular points in  $\varphi'$ . The image  $c'^6$  of  $k^4$  in  $\varphi'$  has triple points in two of these singular points. They are evidently image points of trisecants of  $k^4$ . Accordingly a conic  $k'^2$  of  $R'_3$  is associated to  $\omega^3$ .

The other four singular points in  $\varphi'$ , which are double points for  $c'^6$  and single points for the image  $c'^2$  of  $k^2$ , are evidently image points of lines that cut  $k^2$  once and  $k^4$  twice. To  $\omega^6$  there corresponds, therefore, a biquadratic curve  $k'^4$ . This curve is rational, as the same holds good for  $\omega^6$ , as appears from a plane section, which has four double points and two triple points. The four trisecants of  $k^4$  that can be drawn through the points of intersection of  $k^4$  and  $k^2$ , are common lines of  $\omega^6$  and  $\omega^2$ . Consequently the curves  $k'^2$  and  $k'^4$  have four points in common.

Accordingly to a point of  $k^2$ , through which there pass three lines of

$\omega^6$ , there corresponds a trisecant of  $k'^4$ . To a point of  $k^4$ , through which there pass two lines of  $\omega^6$  and one line of  $\omega^2$ , a line is associated that cuts  $k'^4$  twice and  $k'^2$  once. We find, therefore, that to the points of  $k^2$  the lines of a scroll  $\omega'^2$ , to the points of  $k^4$  the generatrices of a scroll  $\omega'^6$  are associated.

As a line  $r'$  of  $R'_3$  cuts six generatrices of  $\omega'^6$ , any curve  $r^3$  of  $R_3$  has six points in common with  $k^4$ . In the same way it is evident that any curve  $r'^3$  of  $R'_3$  cuts the curve  $k'^4$  six times.

§ 5. Among the transformations derived in § 4 we have, according to § 2, only to consider those for which  $k'^2$  is a singular curve. Such a transformation can be derived from an arbitrary correspondence indicated in § 4, by combining it with one of the  $\infty^7$  collinear transformations of  $R'_3$  in itself which transform  $k'^2$  into  $k'^2$ . To a point  $P$  of  $R_3$  we therefore associate the point  $P'$  of  $R'_3$  which in the chosen collinear correspondence corresponds to the point that is associated to  $P$  through the former transformation.

Now we have in  $R'_3$  two singular curves  $k'^2$  and  $k'^4$  to which there correspond resp. the scrolls  $\omega^2$  and  $\omega^6$ ; to the singular curves  $k^2$  and  $k^4$  in  $R_3$  resp. the surface  $\omega'^2$  of the trisecants of  $k'^4$  and the surface  $\omega'^6$  of the bisecants of  $k'^4$  that cut  $k'^2$  are associated.

Let us now consider the correspondence between the linear complexes  $C$  and  $C'$  that arises if to any line  $l$  of  $C$  we associate that line  $l'$  of  $C'$  of which the image point  $P'$  corresponds in the aforesaid transformation to the image point  $P$  of  $l$ .

In the correspondence found in this way, to a line  $l$  of  $C$  cutting  $l_1$  there corresponds one line  $l'$  of  $C'$ , although to the image point  $P$  on  $k^2$  of  $l$  a line of points  $P'$  in  $R'_3$  is associated.

For the plane pencils of  $C$  containing  $l$  form a special linear congruence which is represented on a plane  $\varphi$  that touches  $k^2$  in the image point  $P$  of  $l$ . The image lines of the said plane pencils are, therefore, the lines of  $\varphi$  through  $P$ . The image curves in  $R'_3$  of these lines all pass through the vertex  $P'$  of the sheaf of planes  $\varphi'$  that correspond to the net of surfaces  $\varphi^3$  which touch  $\varphi$  at  $P$ . This point  $P'$  is the conical point of the surface  $\varphi'^3$  on which  $\varphi$  is represented. For the intersection of any plane through  $P'$  with this surface has a double point in  $P'$  as  $P$  is a double point for the intersection of  $\varphi$  with any surface  $\varphi^3$  that touches  $\varphi$  at  $P$ .

A scroll of  $C$  containing  $l$  is represented on a curve that cuts  $k^2$  at  $P$ . The tangent to this curve at  $P$  lies in  $\varphi$  because the plane pencil through a generatrix of the scroll that has a line in common with the plane pencil  $ll_1$ , tends to a plane pencil which contains  $l$  when the generatrix tends to  $l$ . Hence the curve in  $R'_3$  that is associated to the image curve of the scroll, passes through the point  $P'$  which we derived from the image point  $P$  of  $l$ .

Consequently the scrolls of  $C'$  that are associated to the scrolls of  $C$  which pass through a line  $l$  cutting  $l_1$ , have one definite line  $l'$  of  $C'$  in common. To a line  $l$  of  $C$  cutting  $l_1$  there corresponds, therefore, one line  $l'$  of  $C'$ .

In the same way it is evident that one line  $l$  of  $C$  is associated to a line  $l'$  of  $C'$  which cuts  $l'_1$ .

To the line  $l_1$  of  $C$  the line  $l'_1$  of  $C'$  is associated. From the surface  $\varphi^3$  which corresponds to  $a$ ,  $\omega'^2$  splits off so that also  $a'$  corresponds to  $a$ .

§ 6. The rays of  $C$  that are represented on the points of the rational curve  $k^4$  which cuts  $k^2$  four times, are singular for our correspondence. According to § 1 they form a rational scroll  $\varrho^4$  of the fourth degree.

The generatrices of  $\omega^6$  are the images of the plane pencils of  $C$  that contain two lines of  $\varrho^4$ , hence of the plane pencils of  $C$  that have the points of the double curve of  $\varrho^4$  as vertices. According to § 1 the generatrices of these plane pencils form a congruence (3,3) in  $C$ , so that among these plane pencils there are three that have their vertices in a given plane. Consequently the surface  $\varrho^4$  has a cubic nodal curve  $k^3$  and consists, therefore, of the lines of  $C$  that cut  $k^3$  twice.

In the same way  $C'$  contains a rational biquadratic scroll  $\varrho'^4$  of singular rays, which is represented in  $k'^4$  and consists of the lines of  $C'$  that cut a twisted cubic  $k'^3$  twice.

As to a point of  $k^4$  a line is associated that cuts  $k'^2$  once and  $k'^4$  twice, the rays of  $C'$  corresponding to a line of  $\varrho^4$  form a plane pencil of  $C'$  which contains two lines of  $\varrho'^4$ , hence a plane pencil of  $C'$  with vertex on  $k'^3$ . The lines of  $C'$  corresponding to the rays of  $\varrho^4$ , form the congruence  $R'$  (3,3) which is represented on  $\omega'^6$  and has the generatrices of  $\varrho'^4$  as double lines.

The rays of  $C$  corresponding to a line of  $\varrho'^4$  are the generatrices of a plane pencil of  $C$  with vertex on  $k^3$ . The rays of  $C$  associated to the lines of  $\varrho'^4$ , form a congruence  $R$  (3,3). The generatrices of  $\varrho^4$  are double lines of this congruence.

§ 7. A plane pencil of  $C$  is represented on a line  $s$  of  $R_3$  that cuts  $k^2$ . As this has resp. two, two and three points that are not singular for the transformation  $(P, P')$  in common with a surface  $\varrho^3$ , with  $\omega^2$  and with  $\omega^6$ , the curve associated to  $s$  in  $R'_3$  is a conic that cuts  $k'^2$  twice and  $k'^4$  three times.

Accordingly to a plane pencil of  $C$  there corresponds a quadratic scroll of  $C'$  that has three lines in common with  $\varrho'^4$ .

The same holds good for a plane pencil of  $C$  containing  $l_1$ , as to the point of  $k^2$  that is associated to it there corresponds a trisecant of  $k'^4$  which is the image of a quadratic scroll containing  $l'_1$  that contains three lines of  $\varrho'^4$ .

To a plane pencil of  $C$  containing one generatrix of  $q^4$ , a plane pencil of  $C'$  is associated which has one line in common with  $q'^4$ .

A bilinear congruence of  $C$  is represented on a quadratic surface  $\gamma^2$  in  $R_3$  that contains  $k^2$ . As the number of points not singular for the correspondence  $(P, P')$  which this surface has in common with a curve  $r^3$ , a generatrix of  $\omega^2$  and a line of  $\omega^6$ , is resp. equal to four, two and one,  $\gamma^2$  is transformed by the transformation  $(P, P')$  into a biquadratic surface that has  $k'^2$  as double conic and  $k'^4$  as single curve.

*Consequently to a bilinear congruence of  $C$  there corresponds a congruence (2,2) of  $C'$  that contains the lines of  $q'^4$ .*

This holds also good if the bilinear congruence contains  $l_1$ . For in this case it is represented on a plane  $\varphi$  of  $R_3$  that is transformed into a cubic surface  $\varphi'^3$  through  $k'^2$  and  $k'^4$  to which there corresponds a congruence (2,2) in  $C'$  that contains  $l'_1$  and the lines of  $q'^4$ .

A scroll in  $C$  of the degree  $\nu$  that has  $\beta$  lines in common with  $q^4$ , is represented on a curve of  $R_3$  that is of the order  $\nu$ , cuts  $k^2$  in  $\nu$  points and  $k^4$  in  $\beta$  points. With a surface  $\varphi^3$ , with  $\omega^2$  and with  $\omega^6$  this curve has resp.  $2\nu-\beta$ ,  $2\nu-\beta$  and  $3\nu-2\beta$  points in common that are not singular for the correspondence  $(P, P')$ . The curve associated to it in  $R'_3$ , which is of the order  $2\nu-\beta$ , has  $2\nu-\beta$  points in common with  $k'^2$  and  $3\nu-\beta$  points with  $k'^4$ , is the image of a scroll in  $C'$  of the degree  $2\nu-\beta$  which contains  $3\nu-2\beta$  generatrices of  $q'^4$ .

*Accordingly to a scroll in  $C$  of the degree  $\nu$  that has  $\beta$  lines in common with  $q^4$ , there corresponds a scroll in  $C'$  of the degree  $2\nu-\beta$  that contains  $3\nu-2\beta$  generatrices of  $q'^4$ .*

If  $l_1$  is a  $\nu$ -fold line of the scroll chosen in  $C$ , it is represented on a curve of the order  $\nu-\nu$  that cuts  $k^2$  in  $\nu-2\nu$  and  $k^4$  in  $\beta$  points and is, therefore, transformed into a curve in  $R'_3$  of the order  $2\nu-\beta-\nu$  which cuts  $k'^2$  in  $2\nu-\beta-2\nu$  and  $k'^4$  in  $3\nu-2\beta$  points. In this case the scroll is, accordingly, transformed into a scroll in  $C'$  of the degree  $2\nu-\beta$  that contains  $l_1$  as  $\nu$ -fold line and has  $3\nu-2\beta$  lines in common with  $q'^4$ . From this it follows likewise that  $l'_1$  is associated to  $l_1$  in our transformation.

A congruence  $(\mu, \mu)$  of  $C$  containing the generatrices of  $q^4$  as  $\alpha$ -fold lines is represented on a surface in  $R_3$  of the degree  $2\mu$  that has a  $\mu$ -fold conic in  $k^2$  and an  $\alpha$ -fold curve in  $k^4$ . With a curve  $r^3$ , a generatrix of  $\omega^2$  and a generatrix of  $\omega^6$ , this surface has resp.  $4\mu-6\alpha$ ,  $2\mu-3\alpha$  and  $\mu-2\alpha$  points in common that are not singular for the representation. Hence in  $R'_3$  there corresponds to this a surface of the degree  $4\mu-6\alpha$  that has a  $(2\mu-3\alpha)$ -fold curve in  $k'^2$  and a  $(\mu-2\alpha)$ -fold curve in  $k'^4$ . This is the image of a congruence  $(2\mu-3\alpha, 2\mu-3\alpha)$  in  $C'$  of which the lines  $q'^4$  are  $(\mu-2\alpha)$ -fold lines.

*Consequently to a congruence  $(\mu, \mu)$  of  $C$  containing the generatrices of  $q^4$  as  $\alpha$ -fold lines a congruence  $(2\mu-3\alpha, 2\mu-3\alpha)$  is associated which contains the generatrices of  $q'^4$  as  $(\mu-2\alpha)$ -fold lines.*



It is again easily seen that this property holds also good for a congruence of  $C$  of which  $l_1$  is a  $\varrho$ -fold line, and that in this case the corresponding congruence of  $C'$  contains the line  $l'_1$  as a  $\varrho$ -fold line.

§ 8. Among the  $\infty^{16}$  rational biquadratic twisted curves in  $R_3$  there are  $\infty^{12}$  that cut  $k^2$  four times. In  $\infty^{15}$  ways a projective correspondence can be established between the cubic surfaces  $\varphi^3$  containing  $k^2$  and a curve  $k^4$  that cuts  $k^2$  four times, and the planes  $\varphi'$  of  $R'_3$ . In this way we get  $\infty^{27}$  cubic point correspondences between  $R_3$  and  $R'_3$  for which  $k^2$  is a singular curve. For  $\infty^{19}$  of these cubic transformations the singular conic in  $R'_3$  coincides with  $k'^2$ .

Consequently there are  $\infty^{19}$  quadratic birational correspondences between two linear complexes  $C$  and  $C'$  that associate a given line  $l'_1$  of  $C'$  to a likewise given line  $l_1$  of  $C$ . As we can associate any line of  $C'$  to a given line of  $C$ , we find that there exist  $\infty^{22}$  quadratic birational correspondences between two linear complexes.

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**Chemistry.** — *On some derivatives of di-(2-pyridyl)-amine, and on tri-(2-pyridyl)-amine.* By J. P. WIBAUT and G. L. C. LA BASTIDE.  
(Communicated by Prof. A. F. HOLLEMAN.)

(Communicated at the meeting of May 28, 1927).

1. *Introduction.* Of diphenylamine ( $C_6H_5$ )<sub>2</sub>NH several derivatives are known in which nitro groups or halogen atoms are substituted for one or more hydrogen atoms of the benzene nuclei. Nitroderivatives of diphenylamine may, in general, be prepared by bringing halogen-nitrobenzenes in reaction with aniline, in which a diphenylamine derivative is formed and hydrogen halide is split off. Also the nitration of diphenylamine is possible.

The analogue of diphenylamine in the pyridine series, di-(2-pyridyl)-amine ( $C_5H_4N$ )<sub>2</sub>NH is now also known. Our purpose was to prepare substitution products for this substance, and if possible, to find the constitution of the substances obtained. We have examined the nitration and bromination of dipyridylamine, and have obtained a dinitro- and a dibromo-derivative. Accordingly the hydrogen atoms of the pyridine nuclei in dipyridylamine may be readily substituted, just as this is the case in 2-amino-pyridine itself.

The determination of the constitution of these substances has not succeeded, as unexpected difficulties presented themselves. Our researches in this direction have led, however, to the synthesis of the hitherto unknown tri-(2-pyridyl)-amine, ( $C_5H_4N$ )<sub>3</sub>N.

2-dipyridylamine was prepared for the first time by TSCHITSCHIBABIN, who for this purpose heated 2-aminopyridine and 2-chloropyridine in the presence of chloride of zinc <sup>1)</sup>. STEINHAUSER and DIEPOLDER <sup>2)</sup> carry out this reaction by heating with barium oxide. TSCHITSCHIBABIN has also prepared dipyridylamine by heating 2-aminopyridine at 240—250° for a long time with an equivalent quantity of the hydrochloric acid salt of this base, a method which is quite analogous to the preparation of diphenylamine.

According to our experience this last method is best adapted to the preparation of dipyridylamine, though we have not succeeded in raising the yield above 25 %. We have also heated the sodium compound of 2-aminopyridine at 170—180° with 2-chloropyridine and parafin oil as diluent; the yield of dipyridylamine then amounts to 40 %, but this method of preparation is more laborious, as first the sodium compound has to be made. For the melting point of the dipyridylamine purified by recrystallisation from diluted alcohol, we found 95.5—96°.

<sup>1)</sup> J. Soc. Phys. Chem. Russe, **46**, 1216 (1914).

<sup>2)</sup> J. prakt. Chem., (2) **93**, 392 (1916).

### Nitration of dipyridylamine.

If this base is dissolved in strong sulphuric acid, and if at 0° C. nitric acid of density 1.4 is added, no reaction takes place. We have not been able to detect any formation of a nitramine, which is formed from 2-aminopyridine under these circumstances. The nitration, however, proceeds smoothly, if the mixture of dipyridylamine, concentrated sulphuric acid and nitric acid is heated on the water bath.

It appeared from the analysis of the nitration product that this substance contained two nitro groups. After repeated crystallisation from toluene the dinitrodipyridylamine was obtained in the form of yellow-brown needles of a melting-point of 195—196°.

This substance has a very feebly basic character; it dissolves in strong acids, but is again precipitated on dilution with water. It is very sparingly soluble in alcohol and water.

#### Analysis:

Found C 45.74%; H 2.80%; N 26.98%.

Calculated for  $C_{10}H_7N_5O_4$  C 45.96%; H 2.70%; N 26.82%.

#### Bromination of Dipyridylamine.

To a diluted solution of this base in glacial acetic acid a solution of bromine in glacial acetic was added gradually, care being taken that it was properly cooled. The quantity of bromine added was calculated for the introduction of two bromine atoms. From the reaction product a substance was isolated, which appeared to be a salt with hydrogen bromide and which was obtained after repeated recrystallisation from alcohol in the form of yellow orange needles melting at 253—254°.

#### Analysis:

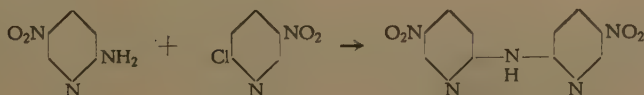
Found 64.99% Br; 8.47% N.

Calculated for  $C_{10}H_7N_3Br_2 \cdot 2HBr$  65.37% Br; 8.56% N.

Accordingly this substance is a salt of dibromodipyridylamine with two molecules of hydrobromic-acid. The alcoholic solution of this salt was made alkaline, whereby the free base was obtained, which after recrystallisation from alcohol melts at 191°; white needles.

Analysis: Br found 48.27% Calculated for  $C_{10}H_7N_3Br_2$  48.63%.

In order to determine the positions which the nitro-groups, respectively the bromine atoms, occupy in these substances, we have tried to prepare these compounds in another way, i.e. by bringing substituted 2-aminopyridines in reaction with 2-halogenopyridines. From 2-amino-5-nitropyridine and 2-chloro-5-nitropyridine 5,5'-dinitrodipyridylamine might be formed:



The synthesis of monosubstitution products of dipyridylamine or of other disubstitution products might be tried in a similar way. We have, therefore, prepared the substituted pyridines suitable for our purpose, and tried to transform them in dipyridylamine-derivatives in the sense of the above-mentioned formula.

The following mixtures of substances were heated with barium oxide, in analogy to DIEPOLDER's method of preparation for dipyridylamine itself:

2-amino-5-nitropyridine	with 2-chloropyridine
"	" 2-bromopyridine
"	" 2-iodopyridine
2-amino-5-bromopyridine	" 2-chloropyridine
"	" 2-bromopyridine
"	" 2-iodopyridine
2-amino-5-bromopyridine	" 2-iodo-5-bromopyridine <sup>1)</sup>
2-amino-3,5-dibromopyridine	" 2-chloropyridine

All these experiments gave negative results. If the temperature was raised to about 150°, no reaction took place, but if the temperature was carried up to 200°, the substances were quite decomposed, only a charry residue being obtained. Nor did TSCHITSCHIBABIN's method for the preparation of dipyridylamine, heating of aminopyridine with an equivalent quantity of the hydrochloric acid salt of this base, give any result for the substituted aminopyridines. Experiments were made with 2-amino-5-nitropyridine and 2-amino-3,5-dibromopyridine, which were each heated with an equivalent quantity of the hydrochloric acid salt. At 150° no reaction took place, at higher temperature again total decomposition.

It appeared, therefore, from these experiments that for the synthesis of substituted dipyridylamines high temperatures are to be avoided, as the substituted aminopyridines are decomposed at increased temperature. Since diphenylamine derivatives may be obtained in many cases from halogennitrobenzenes and aniline by boiling a mixture of these substances in alcoholic solution with addition of sodium acetate, we have tried this method with the following substances:

2-amino-3,5-dibromopyridine	with 2-chloropyridine
2-amino-5-bromopyridine	" 2-chloropyridine
2-amino-5-nitropyridine	" 2-chloro-5-nitropyridine
2-amino-5-nitropyridine	" 2-chloropyridine

The substances, however, do not react under the circumstances of the experiment.

From these experiments we get the impression that a halogen atom bound in the pyridine nucleus in position 2, is difficult to bring in reaction, also

<sup>1)</sup> Instead of with barium oxide we have also heated the substances with potash and copper powder (ULLMANN), likewise however with a negative result.



in those cases where in this pyridine nucleus other halogen atoms or a nitro group have been substituted in position 5.

For these synthetic attempts a number of substituted pyridines had to be prepared, some of which were not yet known. These new compounds will be briefly described here.

*2-iodo-5-bromopyridine.*

This substance was obtained by adding 2-amino-5-bromopyridine dissolved in 50 % acetic acid gradually to a boiling solution of potassium iodide and sodium nitrite, hence the same method being used as for the preparation of 2-iodo-pyridine according to TSCHITSCHIBABIN. 2-iodo-5-bromopyridine crystallizes from alcohol in white plates melting at 117°.

*5-iodo-2-bromopyridine.*

This substance was prepared by treating 2-amino-5-iodopyridine dissolved in strong hydrobromic acid with sodium nitrite. The 2-bromo-5-iodopyridine crystallized from alcohol in shiny plates melting at 122°.5.

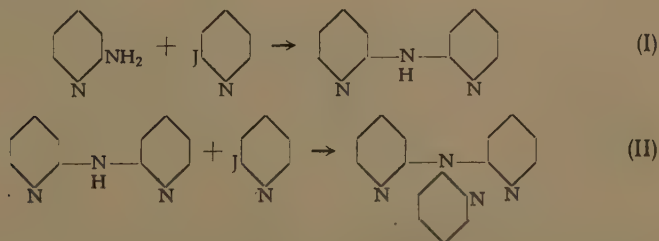
*2-5-di-iodopyridine.*

This substance was obtained in an analogous way from 2-amino-3,5-dibromopyridine. The 2-iodo-3,5-dibromopyridine is obtained from alcohol in white crystal plates, which melt at 70°.5. In appearance and in smell the substance resembles the dihalogenopyridines which have just been described. Experimental details of this experiments and analytical data will be published elsewhere.

3. *Preparation of tri-(2-pyridyl)-amine.*

We have undertaken the synthesis of the still unknown tri-pyridylamine by analogous methods as that tried for dipyridyl derivatives, and have succeeded in preparing this substance in two ways.

A mixture of 2-aminopyridine (0.1 mols.) and 2-iodopyridine (0.2 mols.) dissolved in mesitylene was heated with anhydrous potash, some copper powder and a little potassium during 14 hours at 150—160°; it was found that tripyridylamine had been formed in a yield of 10—20 % of the theoretically calculated quantity. This experiment shows that the method, which was found by ULLMANN for the synthesis of diphenylamine derivatives, may be applied in the pyridine series. The reaction takes place in two stages:



The second reaction proceeds, however, more rapidly than the first, for if only 1 mol. of 2-iodopyridine is brought in reaction with 1 mol. of

aminopyridine in the way just mentioned, only tripyridylamine is formed, and no dipyridylamine can be detected. A good yield of tripyridylamine is obtained when a mixture of dipyridylamine and 2-iodopyridine dissolved in mesitylene is heated with potash and some copper powder and potassium iodide.

On the other hand the sodium compound of dipyridylamine:  $(C_5H_4N)_2N.Na$  does not react with chloropyridine. This result seems plausible, as in sodium amide it are the hydrogen atoms, and not the sodium atom which are reactive, as TITHERLEY has pointed out <sup>1)</sup>. This investigator found in his researches on reactions with sodium amide that this substance does not react with alkylhaloids to form primary amines. From hexachloroethane and sodiumamide was formed among other substances the sodium compound of cyanamide  $CN.N.Na_2$ , in which the sodium atoms are, still present. In TSCHITSCHIBABIN's amidation reaction it is again the hydrogen atom of the sodiumamide which is reactive:



We already mentioned that the sodium compound of aminopyridine  $C_5H_4N.NHNa$  reacts on chloropyridine with formation of dipyridylamine. It is probable that here primarily the sodium compound of dipyridylamine is formed, from which after decomposition with diluted hydrochloric acid, the dipyridylamine itself is obtained. In the sodium compound of dipyridylamine however, no reactive hydrogen atom is available, so that a reaction with chloropyridine cannot take place.

*Tri(2-pyridyl)-amine*  $(C_5H_4N)_3N$ .

This substance may be obtained from diluted alcohol or from a mixture of benzene and ligroin in the form of long colourless needles, which melt at 132.5°.

Analysis:

Found: C 72.51 %; 72.40 %; H 5.08 %; 4.71 %; N 22.67 %; 22.62 %.

Calculated for:  $C_{15}H_{12}N_4$ : C 72.58 %; H 4.88 %; N 22.58 %.

Molecular weight found cryoscopically in benzene solution: 234 and 237, calculated 248.

The picrate of this base crystallises from alcohol in yellow needles, which melt at 149°. Nitrogen content found 20.74 %, calculated for  $C_{15}H_{12}N_4 + C_6H_2OH(NO_2)_3$ : 20.54 %. In the picrate 1 mol. of picric acid is, therefore, bound to 1 mol. of tripyridylamine.

The double salt with mercury chloride crystallises from diluted hydrochloric acid in white needles, which melt at 180°.

N found: 10.16 % and 10.21 %

Calculated for  $C_{15}H_{12}N_4 \cdot HCl \cdot HgCl_2$  10.07 % N.

Tripyridylamine appeared to be a very feeble base, which is little soluble in water. This aqueous solution is scarcely basic to litmus. The compound

<sup>1)</sup> Journ. Chem. Soc., 71, 460.

is readily soluble in diluted mineral acids. The slight basicity of this pyridine base is surprising. Pyridine itself is a basic substance, it might be expected that on substitution of the hydrogen atoms of ammonia by pyridyl groups a base would be formed, which would be comparable in strength to the aliphatic amines. This is, however, not the case. To get an idea of the difference in basicity between 2-aminopyridine, di(2-pyridyl)-amine, and tri(2-pyridyl)-amine we have measured the electric conductivity of 0.01 normal solutions of these bases in a mixture of water and 10 % alcohol. It appeared that dipyridylamine has a greater conductivity than aminopyridine, tripyridylamine, however, shows a smaller conductive power than aminopyridine. This result was confirmed by measurement of the concentration of the hydrogen ions in 0.01 normal solutions in 15 % alcohol, by the aid of the chinhydrone electrode <sup>1)</sup>.

The values found are :

2-aminopyridine  $P_H = 9.07$ .

di(2-pyridyl)amine  $P_H = 9.41$ .

tri(2-pyridyl)amine  $P_H = 7.40$ .

It appears, therefore, that the basicity of aminopyridine slightly increases when a second pyridyl group is attached to the nitrogen atom of the ammonia rest as might be expected, but that the basicity appreciably decreases if the third hydrogen atom of the ammonia rest is substituted by a pyridyl group.

More detailed experimental data of this investigation here will be published elsewhere.

*Organic Chemical Laboratory  
of the University.*

*Amsterdam, May 1927.*

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<sup>1)</sup> The measurements of the conductivity have been made by one of us (LA BASTIDE) in the Electrochemical Laboratory of the University of Amsterdam under the direction of Prof. Dr. A. H. W. ATEN, the  $P_H$  measurements in the Physiological Laboratory of the State University of Groningen, under the direction of Dr. BRINKMAN. We gladly express our indebtedness to both gentlemen.

**Mathematics.** — *On the foundations of combinatory Analysis Situs Additions and Corrections.*<sup>1)</sup> By M. H. A. NEWMAN. (Communicated by Prof. L. E. J. BROUWER).

(Communicated at the meeting of May 28, 1927).

*FI.*

p. 612, l. 16. *Definition of  $\Gamma + \Delta + \dots$ .* This definition does not follow current combinatory usage, which would require: " $\Gamma + \Delta + \dots$  is the  $n$ -array whose units are all the units belonging to an odd number of the arrays". The application contemplated for the results of *FI* and *FII* is, however, rather the combinatory characterisation of continuous spaces than the study of oriented spreads on a complex, and for this purpose the definition of *FI* is the more suitable. There seems justification for adopting the two definitions side by side. Following the notation for sets of points introduced by CARATHEODORY<sup>2)</sup> and adopted by HAUSDORFF<sup>3)</sup> it is natural to denote by  $\Gamma + \Delta + \dots$  the *logical sum* considered in *FI*, i. e. the array whose units are all the units of  $\Gamma, \Delta, \dots$ ; and by  $\Gamma + \Delta + \dots$  the *sum mod. 2*, the array whose units are those belonging to an odd number of the arrays. When no two of the arrays have a common unit we may speak simply of the *sum*, and denote it by  $\Gamma + \Delta \dots$ .

A corresponding duality will appear in the definition of "boundary". It seems reasonable to retain for the word "boundary" the meaning attributed to it in *FI*, viz. the set of faces belonging to only one unit, and to denote this by  $\bar{\Gamma}$ . The sum, mod 2, of the boundaries of the units of  $\Gamma$  will be called the *margin* of  $\Gamma$  and denoted by  $\widehat{\Gamma}$ . Boundary and margin are identical when  $\Gamma$  is regular.

It will be found that *FI* 1 and *FI* 3, the only theorems in *FI* and *FII* affected by this dual definition, have each two forms, viz:

I 1a. If  $\Gamma$  and  $\Delta$  have no common vertex and  $d(\Gamma\Delta) > 0$ ,  $\overline{\Gamma\Delta}$  is  $\Gamma\bar{\Delta} + \bar{\Gamma}\Delta$ .

I 1b. In the conditions of I 1a,  $\widehat{\Gamma\Delta}$  is  $\widehat{\Gamma}\bar{\Delta} + \bar{\Gamma}\widehat{\Delta}$ .

I 3a. ( $n \equiv 2$ ) If  $\Gamma_n$  is bounded and regular,  $\bar{\Gamma}_n$  is unbounded.

I 3b. ( $n \equiv 2$ ) If  $\Gamma_n$  is any array,  $\widehat{\Gamma}_n$  is marginless if it exists.

p. 612, l. 25. For only read at most.

p. 615. *Topological equivalence.* That the three moves 1, 2 and 3 give an adequate account of the relation of homeomorphism between manifolds is an unproven but plausible hypothesis; but for more general arrays it is easy to see that *equivalence*, exhibited by the three moves, and *homeomorphism*, based on (1.1) continuous correspondence (if the units are regarded as continuous simplexes) are not always the same. Moreover there are many purely combinatory problems for which the use of general "spherical" cells is essential. It therefore seems better to regard the theory of arrays as a piece of apparatus

<sup>1)</sup> The original articles appeared in these Proceedings, 29, (1926), 611 and 627. They are here quoted as *FI* and *FII*.

<sup>2)</sup> *Reelle Funktionen*, p. 28.

<sup>3)</sup> *Mengenlehre*, (1927).



for proving theorems about manifolds regarded as "complexes", and specially about spheres and elements. The symbol " $\Gamma \rightarrow \Delta$ ", standing, when  $\Gamma$  and  $\Delta$  are bounded, for  $\Gamma \xrightarrow{1,23} \Delta$  should therefore no longer be read as " $\Gamma$  is topologically equivalent to  $\Delta$ ", but as some conventional phrase, e.g. " $\Gamma$  leads to  $\Delta$ ". In the case of unbounded arrays it is natural from this point of view to restrict " $\Gamma \rightarrow \Delta$ " to the meaning " $\Gamma \xrightarrow{3} \Delta$ " instead of that given on p. 615 of *FI*. When  $\Gamma$  and  $\Delta$  are manifolds this is equivalent to the old definition in virtue of *FII* Theorem 5; but that theorem must now be enunciated:

*If  $S$  and  $T$  are units of the unbounded manifolds  $\Lambda$  and  $M$  respectively, and if  $\Lambda - S \rightarrow M - T$ , then  $\Lambda \rightarrow M$ .*

It will be found that " $\rightarrow$ " without a suffix is rarely used in *FI*, or in *FII* before Theorem 5, so that the necessary verbal alterations are few.

A systematic account of the relations between the theory of arrays and the ordinary theory of complexes will be included in a paper to be published shortly in *Mathematische Annalen*.

p. 615, l. 5 from bottom. The statement that the relation " $\rightarrow$ " is transitive for unbounded arrays was incorrect with the old definition, but as only manifolds were considered no false deductions were made from it. With the new definition (" $\rightarrow$ " means " $\xrightarrow{3}$ " for unbounded arrays) the statement is true.

p. 620. With the new definition I 21 must read: *If  $M$  is an unbounded manifold and  $M \rightarrow \Lambda$ , then if  $S$  and  $T$  are any units of  $M$  and  $\Lambda$  respectively,  $M - S \rightarrow \Lambda - T$ .*

p. 624. l. 10 from bottom. For  $S, \bar{T}'$  read  $\xi, \bar{S}, \bar{T}'$ .

l. 9 " " For  $UV, \bar{T}$  read  $\xi, \bar{UV}, \bar{T}'$  (twice). The proof of I 38 is simplified a little by noticing that since  $W$  is supposed not to belong to a common face of  $M$  and  $\xi S, \bar{T}'$  it must belong to  $T' \cdot \bar{S}$ , i.e. to  $T'U, \bar{V} + T'V, \bar{U}$ .

p. 626 l. 4 from bottom. Delete or is contained in.

## F II.

p. 627, l. 11. "Whether the first of these assertions is true I do not know". The following considerations, based on a theorem proved in a recently published paper<sup>1)</sup>, shew that the conjecture (1) must be rejected; i.e. there are in fact manifolds  $\Lambda, M$  with congruent boundaries, such that  $\Lambda \rightarrow M$ , but not  $\Lambda \xrightarrow{3} M$ .

Let  $\Lambda$  and  $M$  be two manifolds such that  $\Lambda \rightarrow M$ . Since moves of type 3 do not affect the boundary, any chain of such moves relating  $\Lambda$  to  $M$  correlates each bounding  $(n-1)$ -manifold of  $\Lambda$  with a congruent one of  $M$ . Suppose now that  $\Lambda$  has  $q$  bounding manifolds  $\Lambda^1, \Lambda^2, \dots, \Lambda^q$ , all congruent to each other, and let the boundary manifolds of  $M$  be arbitrarily numbered  $M^1, M^2, \dots, M^q$ . When this has been done  $\Lambda$  and  $M$  can be modified into  $\Lambda^*$  and  $M^*$  (e.g. using *FII* Theorem 8a) so that while no two of the  $\Lambda^{*i}$ 's are congruent,  $\Lambda^{*i}$  is congruent to  $M^{*i}$  ( $i=1, 2, \dots, q$ ) and  $\Lambda^* \rightarrow M^*$  still. Hence if  $\Lambda^* \xrightarrow{3} M^*$   $\Lambda^{*i}$  and  $M^{*i}$  are related bounding manifolds. Now it is shewn in *S*<sup>2)</sup> that in these

<sup>1)</sup> "On the superposition of  $n$ -dimensional manifolds", *Journal Lond. Math. Soc.*, 2 (1927) 54, referred to as *S*.

<sup>2)</sup> Proof of Theorem 3.

circumstances  $A$  can be superposed<sup>1)</sup> on  $M$  in such a way that  $A^i$  falls on  $M^i$ . If then the hypothesis (I) were true it would follow that given two manifolds  $A$  and  $M$ , each with  $q$  congruent bounding manifolds then if  $A \rightarrow M$ ,  $A$  can be superposed on  $M$  so that assigned pairs of boundary manifolds coincide. This is clearly false. Example. If  $M_3$  is the finite part of Euclidian space cut out by (a) two "concentric" ring surfaces, <sup>2)</sup>  $R_1^1$  and  $R_2^2$ , and (b) a third ring  $R_3^3$  which is homotope to a point in the finite space cut out by  $R_1^1$  and  $R_2^2$  alone, then  $M_3$  cannot be superposed on itself so that  $R_1^1$  falls on  $R_2^2$ , for  $R_3^3$  is distinguished by the property that it can be separated from  $R_1^1$  and  $R_2^2$  by a sphere lying in  $M_3$ .

It appears then, that the theorems 1, 3a, 3b, and 5 of FII express between them all the relations that exist between the three moves when applied to general manifolds.

p. 627, l. 13. "If  $n > 2 \dots$ " A 3-dimensional example in support of this statement has since been given in these Proceedings<sup>3)</sup>.

p. 630, l. 21. For  $\Lambda - \bar{S} \cdot T + \bar{T} \cdot S$  read  $(\Lambda - S \cdot \bar{T}) + T \cdot \bar{S}$ .

p. 631. The proof of Theorem 2 is obscure. Substitute this for the second paragraph: Consider first the case where  $\Gamma$  contains only one intrusive component,  $V$ . Let the original series of moves  $(\Lambda \xrightarrow{3} M)$  be applied to  $\Gamma + \Lambda$  until a move — say  $U \cdot \bar{V} \rightarrow V \cdot \bar{U}$  — involving  $V$  is reached, and then let  $U \cdot \bar{V}$  be changed to  $\alpha \cdot \bar{U} \cdot \bar{V}$ ,  $\alpha$  being new. Now resume the original series  $(\Lambda \xrightarrow{3} M)$ , with this modification, that all units containing  $V$  are to appear "subdivided through  $V$  with vertex  $\alpha$ ". (See FII p. 630) This modified process can be carried out by moves of type 3. For if  $PQ \cdot RS \rightarrow RS \cdot \bar{PQ}$  is a move of the old series, (where  $PR$  is  $V$ ), and  $\Gamma + \Lambda^*$  is the manifold that has been reached from  $\Gamma + \Lambda$  when this move falls due, the interchange of the corresponding "broken" clusters can be effected provided  $\Gamma + \Lambda^*$  contains neither  $\alpha S$  nor  $RS$  (Lemma 3, with FI 17 and 29). Now  $RS$  does not belong to  $\Lambda^*$  for if it did it would invalidate  $PQ \cdot \bar{RS} \rightarrow RS \cdot \bar{PQ}$  in the original series; and if it were interior to  $\Gamma$  it would be another intrusive component not contained in  $V$ , contrary to the assumption. The  $\alpha$ -star in  $\Gamma + \Lambda^*$  has the form  $\alpha \cdot \bar{V} \cdot \Pi$ , and so the presence of a component  $\alpha S$  ( $S$  not being contained in  $V$ ) would imply the presence of  $\alpha \cdot \bar{V} \cdot S$ , and therefore, if  $V$  is not identical with  $R$ , of  $RS$ , in  $\Gamma + \Lambda^*$ . Thus neither  $RS$  nor  $\alpha S$  is present: the substitution can be effected. If  $V$  is  $R$  I 36 shews that only the condition "not  $\alpha S$ " is necessary in Lemma 3.

p. 632, l. 4 of second par. For belongs to read is interior to.

l. 4 from bottom. For  $E^*$  read  $\bar{E}^*$ .

p. 634. Theorem 6 Corollary 1. To see that  $E_n^1 + E_n^2$  is an  $n$ -manifold assume the corollary to be true for elements of lower dimension-number.

Lemma 5a. For  $\bar{M}$  read  $M$ .

p. 636, l. 2. For  $E_n$  read  $\bar{E}_n$ .

<sup>1)</sup> See S. The word itself probably conveys its meaning accurately enough for present purposes.

<sup>2)</sup> I.e. rings generated by the rotation of two concentric coplanar circles about the same axis in their plane.

<sup>3)</sup> "A property of 2-dimensional elements", these Proceedings, 29 (1927) 1401.

§ 9 (which should be § 4). A generalised and simplified theory superseding § 9 will be published shortly. The restriction on "normal sets" that the common part of  $G_i$  and  $G_k$  must be just one "cell" is not essential. Instead of "normal  $n$ -sets of cells", "assemblies of pieces" are considered satisfying the following conditions: the "pieces",  $G_i$ , are elements, of dimension numbers from 0 to  $n$  and

$P(i)$  if  $G_i$  contains a unit or internal component of  $G_j$ ,  $\overline{G_i}$  contains  $G_j$  unless  $G_i$  and  $G_j$  are identical;

$P(ii)$  if  $i > 0$  every unit of  $\overline{G_i}$  belongs to an  $(i-1)$ -dimensional piece;

$P(iii)$  if  $i < n$   $G_i$  is contained in an  $(i+1)$ -dimensional piece.

Retaining the definition of similarity of *structure* given in *FII* (p. 637) (where "piece" must be set for "cell") it can be shewn that if  $\Gamma$  and  $\Gamma'$  are  $n$ -assemblies of pieces with the same structure and the sum  $\Gamma$  of the  $n$ -pieces of  $\Gamma$  is an  $n$ -manifold, then the sum  $\Gamma'$  of the  $n$ -pieces of  $\Gamma'$  is an  $n$ -manifold, and  $\Gamma \rightarrow \Gamma'$ .

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**Botany.** — *The Influence of Temperature on the Respiration of Phycomyces Blakesleeanus Burgeff.* By S. R. DE BOER. (Communicated by Prof. F. A. F. C. WENT.)

(Communicated at the meeting of June 25, 1927).

FERNANDES, in 1923 described a new method to determine simultaneously the amount of oxygen taken in and the amount of carbondioxide given off during respiration. In this publication (2) however, he did not study especially the respiratory quotients. In order to adapt FERNANDES' apparatus to the study of respiratory quotients in the metabolic gas exchange of fungi, certain alterations had to be made.

The organism studied in these investigations was a + strain of *Phycomyces Blakesleeanus* which was cultivated on starch media (bread) or on media rich in oil (ground linseed). The culture medium was spread out in a thin layer on rough linen which was stretched over a small glass frame in such fashion that the medium was in contact with the air both above and below.

On media rich in oil, but for the rest under similar conditions, the respiration is stronger than on the starch media. At 25° C. at a certain stage of development, on 2½ grm. ground linseed the average amount of O<sub>2</sub> taken in is 15 ccm, the CO<sub>2</sub> given off 10 ccm per hour. (The respiratory quotient  $\frac{\text{CO}_2}{\text{O}_2}$  therefore is about 0.67). On the other hand on 2½ grm. bread the O<sub>2</sub> taken in is 5.75 ccm., the CO<sub>2</sub> given off 7.0 ccm. per hour. (A respiratory quotient therefore of about 1.2.)

In 1905 F. F. BLACKMAN (1) published his wellknown theory in which he claims that physiological reactions are influenced by temperature in a similar way as chemical reactions are influenced by temperature; if only the organism is not injured. According to VAN 'T HOFF's law the reaction-velocity increases 2 or 3 times for a rise of ten degrees in temperature. According to BLACKMAN this VAN 'T HOFF's curve could be found by extrapolation from the values obtained after 1, 2, 3 etc. hours.

KUYPER (3) tried to apply BLACKMAN's theory on the respiration of green peas and other seeds. He found, however, that the respiration adapted itself very slowly to a new temperature. He and RUTGERS (4) therefore pointed out that the "zero-hour"-line never can be extrapolated accurately.

The apparent slowness of adaptation to a new temperature is, according to my opinion, the result of a technical mistake. Afterwards I hope to treat this subject more in detail, in this publication I only want to point out that KUYPER used a respiration vessel of about 1250 ccm. and a suction



velocity of only 3 l. per hour. During the first half hour after the temperature has been raised, gas is therefore collected which was still formed at lower temperature.

Further it has lately become evident that there are many objections against the materials used in KUYPER's experiments. STÅLFELT (6) has namely shown that the seed coat offers an obstacle to the diffusion of gas. From the experiments of SIERP (5) it also appears that peas are, physiologically, very complex structures.

When the facts named above are taken into account it appears to be worth while to determine whether the respiration will directly adapt itself to a new temperature and, if this proves to be the case, what the course of the "zero-hour"-line will be. We also will investigate the magnitude of the respiratory quotient as a function of temperature.

The following experiments were all carried out with a suction velocity of  $3\frac{1}{2}$  l. per hour and in a respiration vessel of 325 ccm. cubic contents. Gas diffusion was promoted as far as possible by using thin layers of the culture medium.

All experiments were carried out in the "constant" part of the grand period of the respiration.

At first 3 or 4 measurements were carried out at the initial temperature so that the course of the respiration is sufficiently known. The change in temperature of the water in the basin, which contains the apparatus, usually takes place in  $2\frac{1}{2}$ —5 min. As a rule  $22\frac{1}{2}$  min. afterwards the experiments start at the new temperature.

#### *Experiments on a medium rich in oil.*

In the ground linseed on which *Phycomyces* is cultivated carbohydrates are present, partly being formed from the fats by the fungus itself.

When the temperature of a culture of *Phycomyces* on linseed medium is changed from 25° to 10°, 15° or 20°, the respiration decreases. The lower the temperature the stronger this decrease will be. The amount of CO<sub>2</sub> given off is immediately constant and has therefore adapted itself directly to the new temperature. However the amount of O<sub>2</sub> taken in decreases and only becomes constant after a few hours. The respiratory quotient will increase and only reassume its original value after some time. The transition from 25° to 15° is shown in fig. 1a.

The reverse, namely the transition from 15° to 25° is given in fig. 1b. The CO<sub>2</sub> evolved is again immediately constant, but now the amount of O<sub>2</sub> absorbed is lower and therefore the respiratory quotient higher at first. From the fact that the CO<sub>2</sub> given off is constant at once it follows that a temperature of 25° is not yet injurious to the fungus.

As soon as the temperature has a harmful influence on the fungus there will be a decrease in the intensity of the respiration. But if in this case

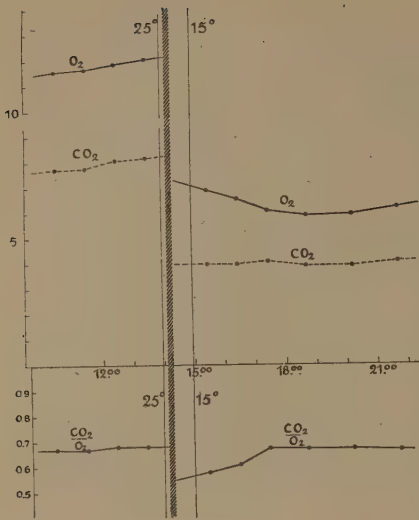


Fig. 1a.

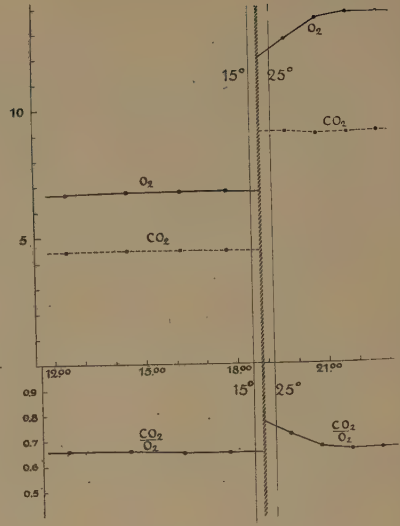


Fig. 1b.

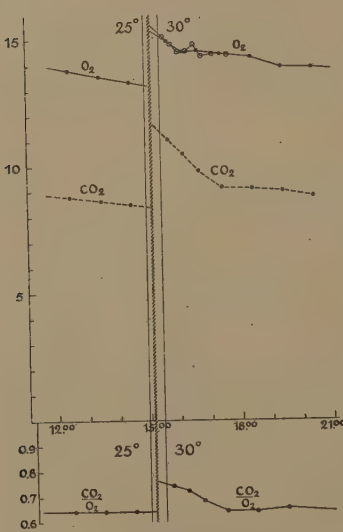


Fig. 1c.

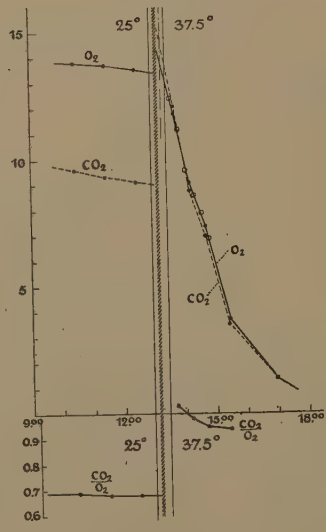


Fig. 1d.

Fig. 1. The respiration of *Phycomyces* on media rich in oil. The ordinate-axis represents the rate of respiration in ccm per hour, the abscissae axis the time in hours. At the same time the ordinate-axis indicates the value of the respiratory quotients. With the changes in temperature the hatched part represents the time required by the water in the basin to take on the new temperature. The vertical line in front of it gives the time at which the last experiment at the initial temperature was finished, the vertical line behind indicates the time at which the new experiments started. For further explanation see text.

too the amount of  $O_2$  absorbed has become smaller as compared with the  $CO_2$  given off, the " $O_2$ -curve" will descend less rapidly than the " $CO_2$ -curve".

This is indeed the case at  $30^\circ$  as is shown in fig. 1c. Here again the respiratory quotient is higher at first, and only reaches its original value when the respiration becomes constant.

The fact, that the respiratory quotients tend to reach a higher value, becomes more apparent at higher temperatures. At  $32^\circ.5$  (where the fungus has already been injured to such an extent that the respiration does not become constant) the respiratory quotient at first approaches the value of 1.00 and never again reaches the value of about 0.67 as found at  $25^\circ$  and  $30^\circ$ . At  $35^\circ$  the respiration decreases rapidly. The respiratory quotient is originally 1.00 and remains at about 0.90. The transition from  $25^\circ$  to  $37^\circ.5$  is given in fig. 1d. The amount of  $O_2$  taken in and the amount of  $CO_2$  given off are about the same and both decrease rapidly. The respiratory quotient is about 1.00. It therefore seems that at higher injurious temperatures the fungus can consume carbohydrates better than fats. This causes a peculiar course of the "zero-hour"-line. If one wants to extrapolate the points of the "zero-hour"-line from the values found it is necessary to trace back the different "injury" curves to the time at which the culture medium reached the new temperature. The temperature lag in the culture medium is larger than the temperature lag of the water in the thermostat. Thermoelectrical measurements convinced me that the best point to choose as the zero time for the different temperatures is about 5 to 10 min. after the water in the basin had taken on the new temperature. There is of course some variation in choosing this point but in the main the course of the "zero-hour"-line will be as is given in fig. 3. The " $O_2$ -curve" is an optimum curve while at higher temperatures the  $O_2$  values approach the  $CO_2$  values and therefore decrease. The course of the " $CO_2$ -curve" does not follow the theoretical curve, but perhaps this is also chiefly due to the fact that at higher temperatures carbohydrates are consumed instead of fats.

#### *Experiments on a starch medium.*

When the temperature of a culture of *Phycomyces* grown on a starch medium is changed from  $25^\circ$  to a lower temperature the  $O_2$  absorbed becomes constant directly. The same is the case with the  $CO_2$  given off, at the most it is a bit higher at first and therefore also the respiratory quotient. The transition from  $25^\circ$  to  $15^\circ$  is given in fig. 2a.

The reverse, the transition from  $15^\circ$  to  $25^\circ$  is shown in fig. 2b. Here again it is evident that in contrast to the respiration on media rich in oil, the  $O_2$  absorbed becomes constant directly and that the respiratory quotient remains constant.

There is also little change in the respiratory quotient when the temperature of the cultures is raised above  $25^\circ$ .

Only if the temperature reaches a point where it becomes injurious to the fungus the quotient gradually approaches the value of 1.00.

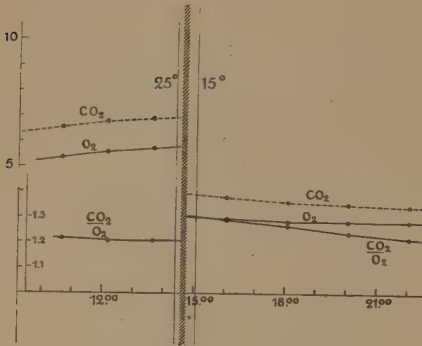


Fig. 2a.

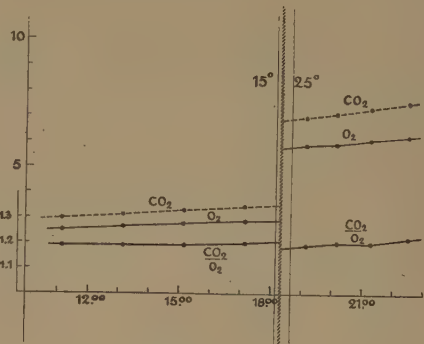


Fig. 2b.

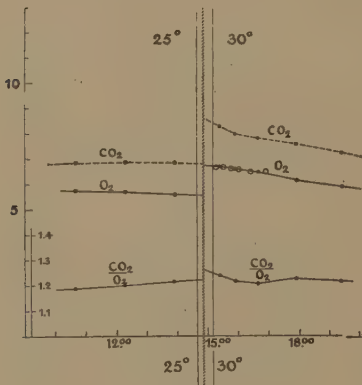


Fig. 2c.

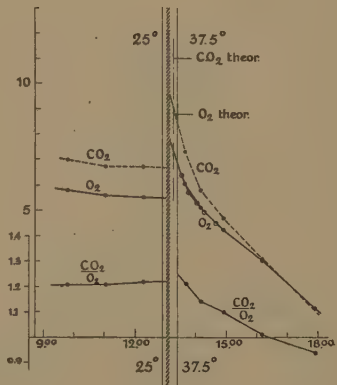


Fig. 2d.

Fig. 2. The respiration of *Phycomyces* on starch media.  
For explanation see text and fig. 1.

The transition from 25° to 30° is given in fig. 2c. At 32°.5, 35° and 37°.5 (fig. 2d) the respiration decreases owing to the noxious action of the temperature. This decrease is, however, much slower than that observed on the fatty media and at 32°.5 finally even becomes constant which is not the case on the oil-media. From this fact it is therefore also evident that on starch media *Phycomyces* can stand higher temperatures better than on oil media.

From the curves given it is obvious enough that the respiration adapts itself directly to new temperatures and not gradually as KUYPER and RUTGERS supposed. Whenever the adaptation seems to be gradual, (the



absorption of  $O_2$  on fatty media) there is a definite reason for this behavior.

The "zero-hour"-line was also constructed for starch media (fig. 4).

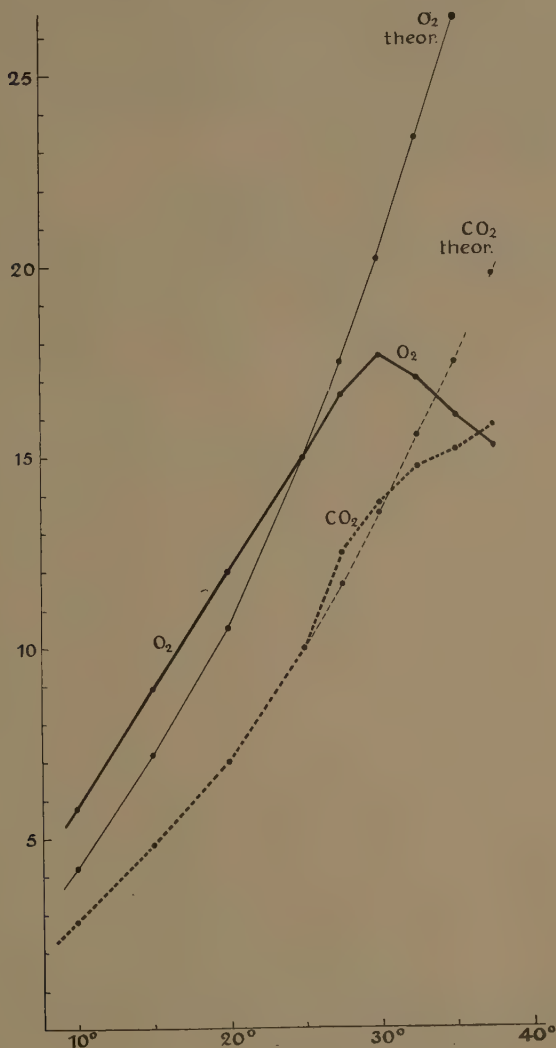


Fig. 3. The respiration on media rich in oil at different temperatures. The ordinate-axis represents the rate of respiration in ccm per hour, the abscissa-axis the temperature in degrees Centigrade. The broad lines represent the course of the "zero-hour"-line, the thin ones give the theoretical values for the higher temperatures. The constant values of the  $O_2$  absorbed below  $25^\circ$ , attained after some time (as explained in the text) are indicated by a thin line.

The curve shows the remarkable fact that the intensity of respiration (as measured by gas-exchange) is an almost linear function of the temperature. The temperature curve as constructed from the data obtained from oil media was slightly convex towards the temperature axis.

The respiratory process on starchy media where  $\text{CO}_2$  and  $\text{O}_2$  respiration run more or less parallel, seems to be less complicated than that on oily media.

The curve seems to suggest that the "zero-hour"-line here also deviates from the theoretical curve at higher temperatures.

A detailed account, dealing with experimental methods and a wider range of external factors, will appear soon.

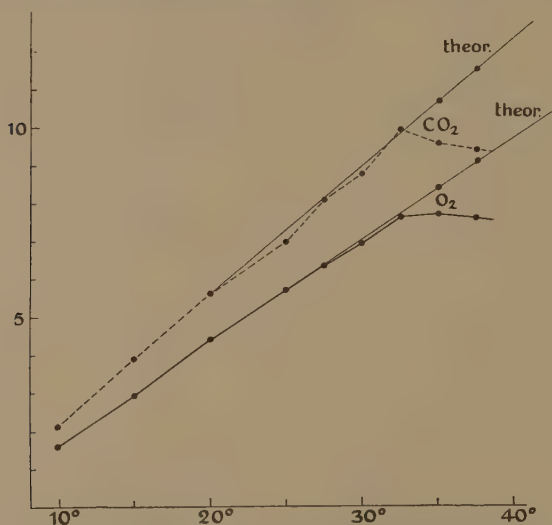


Fig. 4. The respiration on starch media at different temperatures.  
For explanation see text and fig. 3.

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Utrecht, June 1927.

Botanical Laboratory of the University.

**Astronomy.** — *"The proper motions of the globular clusters Messier 13, 56 and 2 and their internal motions."* By ADRIAAN VAN MAANEN.

(Communicated at the meeting of June 25, 1927).

In a previous paper the preliminary results were given of the proper motion of and in the globular cluster, Messier 13; these results were derived from two pairs of plates taken at the Cassegrain focus of the sixty-inch Mount Wilson reflector, taken with intervals of 9 and 11 years, respectively. The present paper discusses the results from two pairs of plates of Messier 56 and one pair of Messier 2, taken with intervals of 9, 11 and 11 years, respectively, at the Cassegrain-focus of the sixty-inch reflector (equivalent focal-length = 80 feet) and from one pair of plates each of Messier 13 and Messier 2, taken with intervals of 14 and 15 years, respectively, at the Newtonian focus of the same instrument (focal-length = 25 feet).

As two of the comparison stars used in the former communication showed a considerable proper motion and two other comparison stars used there could not be measured on the plates taken at the 25-foot focus plates, the former results were reduced with the exclusion of these four stars. The annual proper motion of the members of the clusters is thereby changed very little viz.,  $0''.0015$  in each coordinate.

All plates were measured with the monocular arrangement of the stereo-comparator in four positions, with East, West, North, and South in the direction of the increasing readings of the micrometer screw.

In the case of the first measures of Messier 13, Dr. SHAPLEY had indicated which stars might be members of the cluster and which not. The comparison stars were selected from the latter, the selection being based on distance from the center, magnitude and color-index. For the later measures of Messier 13 and for those of Messier 2 and 56, no such data were available. Accordingly a limiting distance was adopted, outside of which it was assumed that no stars were members of the clusters. The cluster-stars were chosen well within these limits. It is, however, still possible that some of the comparison stars may belong to the cluster, and that some of the cluster-stars are in reality foreground or background stars; but the number, in either case, will probably be small and cannot disturb the mean motions very seriously.

The magnitudes of the stars measured in Messier 13 are due to Dr. SHAPLEY, except for 22 very faint stars measured on the 25-foot focus plates; these were estimated to be between the eighteenth and

nineteenth magnitudes. For Messier 56 the magnitudes were derived by Dr. PANNEKOEK and for Messier 2 by Mr. J. A. BROWN, to both of whom, together with Dr. SHAPLEY, I wish to express my sincere thanks for their kind cooperation.

From the proper motions found for the members of the clusters the following conclusions can be drawn:

A. The relative motion of the three clusters with respect to the mean motion of the comparison stars is:

$$\text{Messier 13: } \mu_{\alpha} = + 0''.0015; \quad \mu_{\delta} = + 0''.0005$$

$$\text{Messier 56: } \mu_{\alpha} = - 0.0018; \quad \mu_{\delta} = + 0.0066$$

$$\text{Messier 2: } \mu_{\alpha} = + 0.0061; \quad \mu_{\delta} = + 0.0042.$$

With the help of Table 26 of "Groningen Publications", N<sup>o</sup>. 29, we can compute the mean parallactic motions of the comparison stars and the corrections from relative to absolute motions of the clusters. These last are then:

$$\text{Messier 13: } \mu_{\alpha} = + 0''.0005; \quad \mu_{\delta} = + 0''.0008$$

$$\text{Messier 56: } \mu_{\alpha} = - 0.0013; \quad \mu_{\delta} = + 0.0066$$

$$\text{Messier 2: } \mu_{\alpha} = + 0.0082; \quad \mu_{\delta} = + 0.0026.$$

Although only three clusters are included in the discussion, it seems worth while to derive from these motions an indication of their mean parallax with the help of the relation between parallax, proper motion, and radial velocity. Using STRÖMBERG's recent value  $\bar{V} = 329$  km/sec. for the mean radial velocity of the globular clusters, the result is:  $\bar{\pi} = 0''.000061$ , which compares favorably with the mean of the three parallaxes found by SHAPLEY, viz.,  $0''.000065$ .

B. The proper motions of the probable members of the clusters were used to compute the probable errors of the motions; for this the deviations of the individual stars from the mean cluster motion were utilized. This gives an upper limit for the probable error, since the internal motions, while small, may still amount to a few thousandths of a second of arc, and further, some stars may have been included which are not members of the clusters. For the plates secured at the Cassegrain-focus we find that the probable error of an individual yearly  $\mu_{\alpha}$  or  $\mu_{\delta}$  derived from one pair of plates is  $0''.0030$ , for the plates secured at the Newtonian-focus,  $0''.0044$ ; this can be expressed better as  $\frac{0''.030}{n}$  and  $\frac{0''.064}{n}$ , respectively, if  $n$  is the interval in years.

C. In "Mount Wilson Contributions", N<sup>o</sup>. 129<sup>1)</sup>, PEASE and SHAPLEY have called attention to the asymmetry found in several globular clusters. While this asymmetry is extremely small, if not negligible, for Messier

<sup>1)</sup> Astrophysical Journal, 45. 225, 1917.

56, they were able to derive "galactic planes" for both Messier 13 and 2. From the flattening of several of the clusters, as well as from theoretical reasons, we might expect the stars in the clusters to be moving in orbits parallel to the galactic planes, around the center of gravity, but according to Russell such motions should be less than  $0''.001$  per year even in the nearest of the clusters. The internal motions for the two clusters, Messier 13 and 2, were analyzed into components, parallel to and at right angles to these planes. In both cases the positive sign was used for motion in the directions southeast and away from the "galactic plane", respectively. The results are:

#### Messier 13

$$\begin{aligned} \text{Two pairs of plates, 80-foot focus} & \left\{ \begin{array}{l} \bar{\mu} // = -0''.0004 \pm 0''.0003 \\ \bar{\mu} \perp = +0''.0005 \pm 0''.0003 \end{array} \right. \\ \text{One pair " " 25-foot " "} & \left\{ \begin{array}{l} \bar{\mu} // = -0''.0001 \pm 0''.0005 \\ \bar{\mu} \perp = -0''.0008 \pm 0''.0005 \end{array} \right. \end{aligned}$$

#### Messier 2

$$\begin{aligned} \text{One pair of plates, 80-foot focus} & \left\{ \begin{array}{l} \bar{\mu} // = -0''.0003 \pm 0''.0002 \\ \bar{\mu} \perp = +0''.0002 \pm 0''.0002 \end{array} \right. \\ \text{One " " " 25-foot " "} & \left\{ \begin{array}{l} \bar{\mu} // = +0''.0004 \pm 0''.0006 \\ \bar{\mu} \perp = -0''.0004 \pm 0''.0005 \end{array} \right. \end{aligned}$$

The dispersion of the motions in the direction parallel to and at right angles to the "galactic plane" is practically the same; this indicates that there is no pronounced motion parallel to the galactic planes.

D. The internal motions for all three clusters were finally analyzed into radial and tangential components, the positive sign being used for motion outward and in the direction **N E S W**.

A radial motion might be expected if the clusters were either expanding or contracting. For Messier 13 we find from the 80-foot focus plates the mean radial motion  $+0''.0006 \pm 0''.0003$ , and from the 25-foot focus plates  $+0''.0013 \pm 0''.0005$ ; for Messier 56,  $-0''.0002 \pm 0''.0005$ ; for Messier 2,  $-0''.0004 \pm 0''.0003$  from the 80-foot focus plates and  $+0''.0019 \pm 0''.0005$  from the 25-foot focus plates. While there is slight preponderance of the positive sign, indicating an expansion of the clusters, the amount is too small to draw any definite conclusions.

From C it follows that the motions resulting from a possible rotation of the clusters are small. Tangential components of the motions were derived therefore only because evidence of such motions had been found in the measures of spiral nebulae. The results for the clusters are:



Messier 13	{two pairs of 80-foot focus plates: $\overline{\mu}\text{rot.} = +0''.0001 \pm 0''.0003$			
	{one pair of 25-foot    "    " :    "    " = $-0.0034 \pm 0.0004$			
Messier 56	two pairs of 80-foot    "    " : $\overline{\mu}\text{rot.} = -0''.0002 \pm 0''.0003$			
Messier 2	{one pair of 80-foot    "    " : $\overline{\mu}\text{rot.} = +0''.0001 \pm 0''.0002$			
	{one pair of 25-foot    "    " :    "    " = $-0.0026 \pm 0.0005$			

In the mean, the tangential component for all the cluster plates taken at the 80-foot focus is exactly  $0''.0000$ , while in the case of Messier 33, observed at that focus, an annual tangential component of  $0''.0140$  was found; for the cluster plates taken at the 25-foot focus the mean tangential component for Messier 13 and 2 is  $0''.0030$ , while for seven spirals it is  $0''.0184$ .

It has been thought that the large displacements in the spirals might be due to a difference in quality in the old and the new plates. In that case the annual motions derived would be smaller for pairs of plates with longer intervals, but the total displacements would be of the same order, and should be the same for both the clusters and the spirals. We find, however, that the total rotational displacement in the case of the 80-foot focus plates is  $0''.070$  for the spirals,  $0''.000$  for the clusters, and, in the case of the 25-foot focus plates,  $0''.174$  for the spirals, and  $0''.044$  for the clusters.

The length of exposure too cannot have caused the displacements in the spirals; for all the 80-foot focus plates for Messier 33, as well as for the clusters, these were about half an hour, except for Messier 2, where the exposure times were 15 and 20 minutes. The exposures for the 25-foot focus plates of the spirals ranged from one hour to eight hours, while for the clusters they were 100 minutes and five hours, respectively.

Since the comparison stars for the spirals are brighter, on the whole, than the nebular points measured, it has been suggested that the rotational displacements found in the spirals might be due to some obscure magnitude error. It was for this reason that several faint objects were measured on the plates of the clusters taken at the 25-foot focus. While no stars fainter than magnitude 16.5 could be measured on the plates taken at the 80-foot focus, the range in magnitude for the 25-foot focus plates was from 12.5 to the eighteenth or nineteenth magnitude for Messier 13, and from 10.3 to well below the seventeenth magnitude for Messier 2. No correlation whatsoever seems to exist between magnitude and tangential component.

The result of the investigation therefore is not only a further proof that the internal motions in the globular clusters are extremely small, but it is also a proof that the displacements found in the spiral nebulae cannot be due to any of the systematic errors discussed above.

*Pasadena, California, U.S.A., May 1927.*

**Anthropology.** — *On the continuance of the Increase of Stature in Holland.* By Prof. A. J. P. VAN DEN BROEK. (Communicated by Prof. L. BOLK.)

(Communicated at the meeting of June 25, 1927).

In 1925 the "Kon. Akademie van Wetenschappen" appointed a committee for the study of the anthropological composition of the Dutch people. Its task comprises, besides a historical review of the Dutch anthropology, an investigation of the stature, the shape of the head, the colour of hair and eyes, and the biological blood-examination.

In the following pages we shall report a few results of the examination of the stature, and chiefly of the phenomenon of the increase in length.

This phenomenon is well-known, BRUINSMA as well as BOLK have given their attention to it in their studies of 1907 and 1910. It is nevertheless of interest to continue the study of this increase of stature, in the first place to see whether it is still proceeding in the same measure as the previous data indicate; secondly to ascertain whether the great war and the consequent troubles in the food-supply has brought any change, and thirdly because the present exhaustive research admits of entering into many details.

We will preface our investigation with a few words about its technique.

The measures have been derived from the registers of the conscripts in the Provincial Registries, i.e. all individual measures were noted down, in order to compute from them the averages.

First of all I feel urged to express my indebtedness to the Provincial Governors, and to the officials involved, for according me the free use of these registers, and also to those who took upon them the laborious work to enter these measures in the proper lists. This is no doubt a cumbrous and a time-consuming procedure, but on the other hand it yields the most accurate results.

The registers alluded to above contain the names of all the 19-year-old men; of nearly all of them the stature has also been given, so that the survey is, indeed, very complete. From these provincial registers the measures have been drawn for every municipality separately, and an average has been computed for each of them. It stands to reason that such an average is only of relative significance for the small municipalities with their small number of conscripts, as in a definite year this average may be considerably influenced by a single very short, or a single very tall individual.

This difficulty has been obviated by taking the averages for a period of 5 years. We chose the period from 1921—1925, in which 285330 measures were entered. The average found for every municipality has been mapped out and has been indicated by different hatching. The two maps accompanying this paper show the result. On the first map we see all statures of 170 cm. and upwards; on the second those under 170 cm.

In this way the statures in the various parts of the country can be surveyed at a glance.

The working of the material, just now described, differs slightly from BRUINSMA's and BOLK's method.

These investigators derived their material from the lists of the Central Office for Statistics, and made groups of certain statures. BRUINSMA distinguishes four groups, viz. less than 155 cm.; 155—159 cm.; 160—169 cm. and upwards of 170 cm.

BOLK made 6 groups, viz. minimal ( $<150$  cm.); very small (150—155 cm.); small (155—160 cm.); middle-sized (160—170 cm.); tall (170—180 cm.) and very tall ( $>180$  cm.).

It is evident that it is rather the procentic relation of the various groups and the shiftings of these relations that find expression in their groups, than the absolute measures that were calculated in the present inquiry.

Still, a comparison is possible.

For the years 1863—1925 BRUINSMA reports the mean statures of the conscripts in periods of 5 years, but he does not say how the averages have been obtained.

BOLK also gives for his six groups a mean stature for the provinces: 148 cm.,  $152\frac{1}{2}$  cm.,  $157\frac{1}{2}$  cm., 165 cm., 175 cm., and 182 cm. This author holds the opinion that the results yielded by these averages can differ only slightly [or not at all] from the actual averages.

He also adds, that "es sich leicht nachweisen lassen würde dass die durch diese Methode gefundene Durchschnittmaasse eher etwas zu klein als zu gross ausfallen müssen". (l. c. page 38).

BRUINSMA points to a detail that of course vitiates his calculations, viz. the fact formerly not all conscripts were measured, but only those that came personally to draw. This leads him to conclude "that the average stature of the measured recruits is on that account smaller than that of all the recruits collectively". This results from the fact that those recruits, whose stature was not up to the mark, came to draw, because they were sure of being rejected, whereas it was just most of the taller ones that stayed away. This holds for the towns far more than for the country.

In later years the measurements were taken at the same time when the recruits were examined medically, so that these data are far more complete. Our results must be considered in connection with the small differences resulting from the different methods of inspection.

The following table shows the average stature of the Dutchman in the periods of which data are at our disposal:



Fig. 1.





Period	Average Stature	Investigator
1863—1867	164.1	BRUINSMA
1867—1872	164.5	"
1872—1877	164.9	"
1877—1882	165.2	"
1882—1887	165.6	"
1887—1892	166.4	"
1892—1897	166	"
1898—1907	168.72 (168.60*)	BOLK
1921—1925	170.77	V. D. BROEK

This table shows at a glance that the stature increases regularly, and that in recent years this process has been progressing. For the first three periods recorded by BRUINSMA the increase is approximately equal, and amounts to 0.8 mm. for each year. Then follows a period of 5 years with an increment of 1.6 mm. per year, succeeded by a period of a slight fall, as had indeed been pointed out by BRUINSMA.

	1898—1907		1921—1925		Increase
	Number	Average*)	Number	Average	
Groningen	25586	169.5	15738	171.47	1.97
Friesland	28268	169.7	16371	172.33	2.63
Drente	12273	167.46	8849	170.80	3.34
Overijssel	27966	168.32	19715	171.05	2.73
Noord-Holland	80857	169.1	51822	171.61	2.51
Zuid-Holland	91485	168.91	70848	171.17	2.26
Utrecht	20145	169.56	14487	171.33	1.77
Gelderland	47116	168.2	31816	170.59	2.39
Zeeland	18741	167.57	10150	169.78	2.21
Noord-Brabant	45070	166.85	28384	168.79	1.94
Limborg	24822	167.48	17150	167.83	0.35
Nederland	422629	168.6	285330	170.77	2.17

\*) These averages have been calculated from the procentic ratios of the various statures, as given by BOLK in the Ned. Tijdschrift voor Geneeskunde 1909.

Putting the lapse of time between the latter results of BRUINSMA, and those of BOLK at 10 years, we get a mean increase of the stature in that period of no less than 2.2 mm. per year.

Between the years 1903 (the average of the period 1898—1907), and the year 1921 the increase amounts to 0.86 mm. per year, and in the latter period of five years it is 1.8 mm. per year.

Before discussing the cause of this phenomenon we have to answer the question whether all the parts of the country have equally contributed to bring about this result. The answer will be found in another table, in which we have tabulated the average statures for the periods 1898—1907 and 1921—1925, together with the number of individuals underlying our calculations. (See table 2.)

This table shows first of all, that the increase of stature is observed in all the provinces, but that great differences occur. I am able to add that the lists of all the municipalities individually show that the increase took place in every one of them. We can, therefore, say that it is universal and is not restricted to a definite class, or district.

It is of interest to establish this, as it follows, that we have not to do only with environmental conditions, or with conditions of welfare.

Various provinces present marked differences. The general average for the whole state is surpassed by the provinces of Friesland, Drente, Overijssel, Noord-Holland, Zuid-Holland, Gelderland, and Zeeland.

It is most striking that the stature increases by as much as 3.34 cm. in the space of 20 years in the province of Drente, as it goes without saying that Drente is not at all the most prosperous, nor the most fertile province.

A look at the map of Drente specially shows, that the distribution of the statures is rather regular. In a small middle-zône from S. E. to N. W., including the municipalities of Steen, Rolde and Vries, the stature is 169 cm., in the largest portion of the province it is 170 cm., while it rises in the South West as high as to 171 cm. The municipalities of Meppel, Nijeveen, and Ruinerwold have, just as Assen, a stature of 172 cm., while only in the Northern municipality of Eelde an average of 173 is attained. These figures serve to bring out the fact that the provincial average does not result from a group of very tall men in a small district, who would be able to markedly affect the general average.

It is of interest now to compare the present results with those obtained by BOLK in 1910. In his study "On the increasing of the stature of the male population of the Netherlands" this investigator reports that between the years 1898 and 1907 the number of minimals fell from 2.3 % to 1 %, while also the number of the very small ones in this province had decreased most, viz. 1.5 %. Compared with other provinces it may be assumed that already in this period the population of Drente was growing faster than that of the other provinces, also when considering that in the said period the population of Drente was the smallest in the Netherlands. In the

P. VAN DEN BROEK: "ON THE CONTINUANCE OF THE INCREASE OF STATURE  
IN HOLLAND".

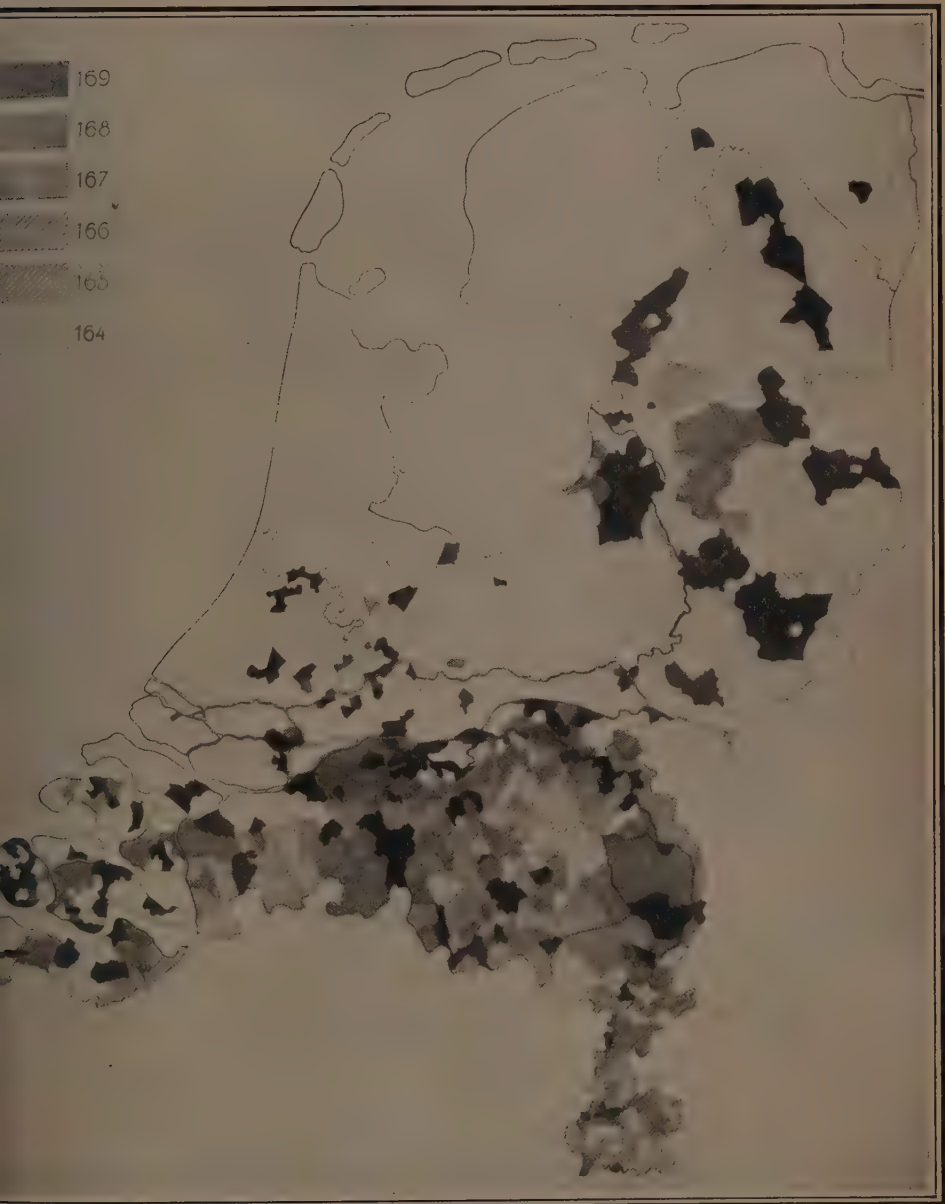


Fig. 2.



period 1898—1907 the average length was in 9 provinces greater than in Drente, and smaller only in one province (Noord-Brabant). In the period 1921—1925 Drente is the 7<sup>th</sup> in order and the average length in the provinces of Gelderland, Zeeland, Noord-Brabant and Limburg is smaller. No doubt a very remarkable improvement in the physical condition, as expressed in the stature.

In establishing the great difference in stature in the period investigated by BOLK (1898—1907), and the one examined by me (1921—1925), one is inclined to think that it results from the difference of the methods. This, however, is not the case, for if it were so, a similar difference would exist also for other provinces in a still higher degree. The exception presented by Drente does not originate in a difference of methods.

However, in order to invalidate this argument completely, I have computed the relations of the period 1916—1925, according to BOLK's division. To the following effect.

In the period 1898—1907 the number of conscripts smaller than 155 cm. amounted to 4.87 %; that of the conscripts taller than 170 cm. was 35.73 % of the total. In the period 1916—1925, 1.35 % of the inspected conscripts were smaller than 155 cm. On the other hand no fewer than 55.68 % were taller than 170 cm.

From this it will be seen, how very large was the shifting to the larger measures in this period.

Does the process of growth in this province come to a standstill?

We are justified in asking this question with a view to the result mentioned in the table on page 693. The increase of stature between the years 1921 and 1925 being 0.24 cm. In the period 1916—1921 it amounted to 1.35 cm. I deem it impossible to fully answer this question on the basis of the data at my disposal at the present moment.

A second question that rises is whether the increase in stature is a phenomenon of the autochthonous population, or whether it has arisen from immigration.

As to this the following remarks can be made.

The increase of population in Drente has been very large in the period from 1830—1920, the largest but one in the whole country. Drente is surpassed in this respect by only one province, viz. Zuid-Holland. Drente does not possess large towns. Zuid-Holland has more of them than any other province. As regards Drente there is no question about migration to the large towns.

The conditions of the soil and of environs are not such as to induce people from Groningen and Friesland to migrate in large numbers to the less prosperous province of Drente. This is why I may be allowed to assume, that it is the autochthonous population that increases in stature. The large birth-rate surplus, which is almost highest in Drente lends support to this assumption.

Overijssel occupies the second place with an increase of 2.73 cm.





Fig. 3.

Parts in which in 1921—1925 the average stature was  $> 170$  cm.

Parts in which in 1898—1907 more than 50% of the conscripts were  $> 170$  cm.  
(after BOLK)

In this province the average statures in the different municipalities present larger divergencies than in the province of Drente. They vary from 165 to 173 cm. The increase of stature in the period between the present data and BOLK's, is virtually the same for all districts. Prof. BOLK's pronouncement of 1909 and 1910 might be applied verbally here. On the accompanying map it can be seen at a glance that the industrial districts and the towns contrast favourably with their surroundings. Hengelo, Enschedé, Oldenzaal, Stad and Ambt-Almelo are so to speak islands with a taller population among the smaller country-people.

The following communication occurring in BOLK's description of his investigation might as well be recorded as a result of the present research. It runs: The smallest population of our country lives in a small zone, proceeding through the middle of Overijsel from the South to the North. This zone includes besides the villages of Laren in Gelderland, the following Overijsel municipalities: Bathmen, Markelo, Holten, Raalte, Dalfsen and Ambt Ommen. However, as soon as we go a little eastward of the zone just described, the population-type changes for the better, as we come in a district poor in undersized recruits and rich in tall and very tall ones. This district includes the municipalities Ambt Almelo, Stad Almelo, Borne, Hengelo, Oldenzaal, Lonneker, and Enschedé.

Of these two groups of municipalities I once more computed the statures in the years 1921 and 1925 separately, to the following effect:

	1921	1925	Increase
Group Laren, etc.	168.15	169.09	0.94
Group Almelo etc.	171.40	172.66	1.26

Indeed, the pronounced difference in stature between the two groups exists also now and even in a higher degree, the growth of the tallest group being slightly more intense. A change from the condition of the beginning of the century is that at present the Overijsel-group of municipalities with the small population no longer belongs to the smallest of the whole country, for already in 1921 they surpassed the averages of the province of Limburg. This could, indeed, be anticipated in view of the pronounced increase of stature in the whole province. The second group of municipalities, the industrial district of Twente, indeed, possesses a population that belongs to the tallest in the Netherlands.

As to Utrecht we call attention to the phenomenon that apparently has existed long, viz. a relative increase of the number of small people. In a total of 14487 men there were in the period 1921—1925 no fewer than 155 shorter than 155 cm. i.e. 1.07 %.

In the period 1898—1907 this relation was only 0.95 %.

Very likely it is this increase of the number of short people that brings down the general increase of stature in this province, which is with 1.77 rather far below the general average in the whole country. The number of tall men ( $> 1.70$ ) has increased from 47.4 % in 1898—1907 to 61.11 % in 1921—1925.

When asking what might be the cause of this remarkable fact, one is reminded of the relatively frequent occurrence of goitre in this province, and one is apt to suppose some causal connection between the two phenomena. Only an extensive investigation of this province, and of other parts of the country where goitre has also prevailed, will enable us to settle this question. In the present paper we only wish to call attention to it.

One word about the province of Noord-Holland :

The increase of the average stature in this province is considerable and agrees about with that in the province of Friesland, Gelderland, Zuid-Holland, and Zeeland. It is expedient to keep Amsterdam apart from the rest of Noord-Holland on account of the large number of Jews, that Amsterdam supplies. On the map, indicating the average stature, it may be seen already that this stature is considerably smaller than in its vicinity. The following table illustrates a detailed study of the increase of stature :

	1898—1907		1921—1925		Increase
	Number	Average	Number	Average	
Amsterdam	41796	168.4	27119	170.8	2.40
N.-Holland without Amsterdam	39061	169.84	24703	172.47	2.63

Two facts are illustrated here. Firstly that the people of Noord-Holland with the exclusion of Amsterdam, are the tallest of the Netherlands, and secondly that the increase of stature outside Amsterdam is a little larger than that in Amsterdam. This difference, however, is so little that it may be disregarded. Also here a detailed investigation of Amsterdam itself, such as K. H. BOUMAN made for the schoolchildren, would yield further results.

Finally the province of Limburg. Here the increase of stature between the periods studied by BOLK and by the present writer amounts to 0.35 cm., far less than in all the other provinces. That this slight increase is not to be considered exclusively as a racial feature, appears directly from a comparison with Noord-Brabant and Zeeland.

True, it may be expected *à priori* that the increase of stature of a short race, i.e. the Alpine, will in a definite space of time be less than in a tall race i.e. the Teutonic, but this by no means accounts for the relation in Limburg. In Noord-Brabant, where the alpine element of the population

is not smaller than in Limburg, the increase of stature is much greater than in this province. Also in Zeeland where the Alpines form a considerable element, the increase of stature is much greater.

An exclusive racial factor, therefore, is hardly to be thought of. After this the idea of immigration forces itself upon us, especially in the mining district. An investigation into the increase of population, the birth-rate surplus in Limburg, does not favour the idea of considerable immigration. For this province also an investigation of longer periods has to be looked out for, before there can be any question of a further pronouncement.

Finally I tabulated the figures for the statures in the years 1921 and 1925 separately.

	1921	1925	Increase
Groningen	170.90	171.60	0.70
Friesland	172.09	172.44	0.35
Drente	170.68	170.92	0.24
Overijssel	170.65	171.51	0.86
Noord-Holland	171.04	171.82	0.78
Zuid-Holland	170.47	171.23	0.76
Utrecht	170.09	171.76	1.67
Gelderland	170.15	170.90	0.75
Zeeland	169.92	170.98	1.96
Noord-Brabant	168.60	168.99	0.39
Limburg	167.56	168.08	0.52
<b>Nederland</b>	<b>170.27</b>	<b>170.97</b>	<b>0.70</b>

This table shows that in the period from 1921—1925 there is an increase of the average stature of 7 mm. This average is surpassed in the provinces of Overijssel, N.-Holland, Z.-Holland, Utrecht, Gelderland and Zeeland. In the others it was lower. — We made the comparison to ascertain whether there was any influence of the war-time with its exceptional feeding-conditions, on the growth of stature. Researches in Germany have made out that the bad feeding-relation in the latter years of the war and the year 1919, have largely influenced the physical conditions of the young people. — The conscripts of the year 1921 were 12 years old in 1914, i.e. the years of the war (and the year 1919) fall for them in their life-period from 12—17; the conscripts of 1925 were in 1914 only 8 years old, so that for them the war-period fell in their life-time from 8—13 years.

If with us the shortage of food had been so great as to be highly detrimental to the growing organism, this would have been evidenced either by an excessive difference between the two groups, or by the absence of such a difference.

Now this is not the case, so that we must conclude that the process of growth as such, has not been influenced by underfeeding during the said years, or that a contingent-deficiency has been compensated later on.

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**Anatomy.** — *Structural parallelism in the midbrain and tweenbrain of teleosts and of birds.* By E. HORNE CRAIGIE Ph.D. (Toronto) and RICHARD M. BRICKNER M.D. (New York). From the Central Institute for Brain Research, Amsterdam. (Communicated by Dr. C. U. ARIËNS KAPPERS).

(Communicated at the meeting of September 24, 1927).

In the course of recent investigations on certain aspects of the brain anatomy of fishes and of birds carried on by the respective writers, (BRICKNER '28, CRAIGIE '28) it became apparent that a remarkable parallelism exists between some of the principal structures of the midbrain and of the 'tweenbrain in the two groups studied. The general phylogenetic relations of these parts are discussed in the paper on the midbrain and thalamus of the teleost (BRICKNER '28), but it appears worth while to consider in a little more detail the analogies observed in the two classes mentioned.

At first sight it may appear to be temerity to compare directly two groups of animals so far apart in the vertebrate series, and especially to do so without a full treatment of the intermediate groups.

There are, however, important considerations which invite such comparison and render it of definite biological interest.

The classes considered are both essentially visual in habit, the sense of sight being one of the most important in each. Moreover, these are the two classes of animals which regularly move in a labile medium without constant solid support, so that the equilibratory sense (with the related one of the lateral line organ in the case of fishes) must also be of great importance. The structures to be discussed, while probably concerned with the correlation of these two functions, have to do mainly with the former one.

Hence it is to be expected that the centres concerned with both these senses will show a more or less parallel development, and, far apart as the animals may be phylogenetically, it is reasonable to look for similarity in structure resulting from convergence in evolution.

The present review, therefore, must not be regarded as an attempt to draw definite homologies, except in the case of the inner segment of the dorsal thalamus as a whole, but rather as a presentation of an interesting series of analogies resulting from functional resemblances, as outlined above.

In the inner part of certain portions of the mesencephalon and thalamus of the fish, there are three main areas, all of which approximate the midline

(Fig. 1). The dorsal one has been termed the *nucleus mesencephali dorsalis*, the middle one the *inner segment of the dorsal thalamus* and the ventral the *nucleus mesencephali ventralis* (BRICKNER '28).

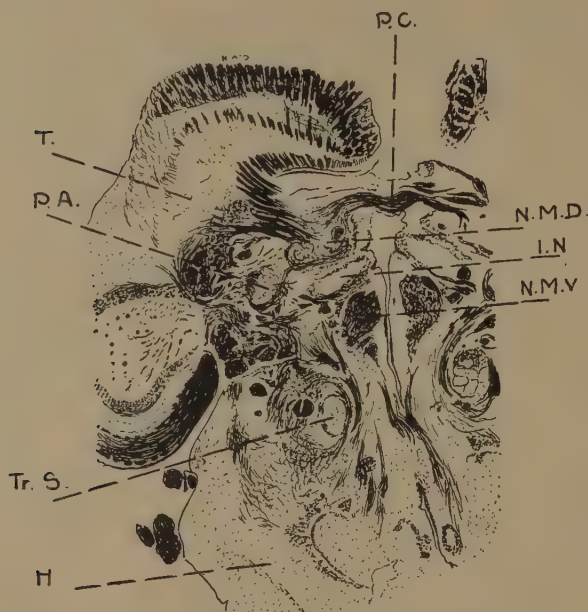


Fig. 1. Transverse section of the brain of *Pleuronectes limanda*.  
(Weigert Pal preparation).

*H.* = hypothalamus; *I.N.* = inner segment of the dorsal thalamus; *N.M.D.* = Nucleus mesencephali dorsalis with fibers entering it from the tectum; *N.M.V.* = Nucleus mesencephali ventralis; *P.A.* = pars anterior of the corpus glomerulosum (Nucleus anterior of authors); *P.C.* = posterior commissure; *T.* = tectum; *Tr. S.* = tractus strio-thalamicus.

The teleostean *inner segment* (*I.N.* Fig. 1, 2, 3) has received its name partly because of its general position in the brain, but especially because it is bounded ventrally by a sulcus, considered comparable (KAPPERS, p. 931, 1921.) to the sulcus medius of amphibia, as described by HERRICK ('17) in his account of the brain of *Necturus*.

It lies (ventro-dorsally) between two tegmental motor areas (nuclei mesencephali dorsalis et ventralis, Fig. 1). It occupies all the transverse levels of the posterior commissure, and extends further, both caudally and frontally, than the latter, reaching as far as the habenula. Its lateral border is difficult to define, but medially it forms a distinct projection into the third ventricle. Its frontal part is limited ventrally by the sulcus just referred to, which is usually quite deep (Fig. 1), but which rarely, if ever,

is present under the caudal portion of the inner segment. The fasciculus retroflexus passes over the area directly caudal to its medial portion.



Fig. 2. Sagittal section of the brain of *Periophthalmus kohlreuteri*.  
(Weigert Pal preparation).

C. = cerebellum; F. = forebrain; F.D. = fasciculus descendens lying in the nucleus mesencephali ventralis; F.R. = fasciculus retroflexus surrounded by medullated fibers running to the nucl. mesencephali dorsalis; H. = hypothalamus; P.C. = fibers to the posterior commissure; P.L.F. = posterior longitudinal fascicle; T. = tectum; Tr.m.v.B. = Tractus mesencephali ventralis bulbaris including a small part of the tr. hypothalamo-bulbaris.

The teleostean inner segment is fairly large, but not nearly so extensive as the avian one. It is not highly differentiated, most of its connections being either completely or almost unmyelinated.

It has not been possible, so far, to divide it into distinct nuclei, or even regions, so that it is also extremely simple in comparison with the inner segment of the bird.

In the fishes, it is connected with the optic nerve, tectum, geniculate body, and nucleus intermedius of GOLDSTEIN (probably part of the geniculatum) by large fibre bundles. Through the latter nucleus, it has further extensive relations with large parts of the visual area. There are a very probable forebrain connection (tractus segmento-interno-telencephalicus), and a definite contingent of fibres relating it to the hypothalamus

(tractus segmento-interno-hypothalamicus, see fig. 3). Whether these various connections are concerned with different sections of the teleostean inner segment, it is, of course, impossible to say at present.

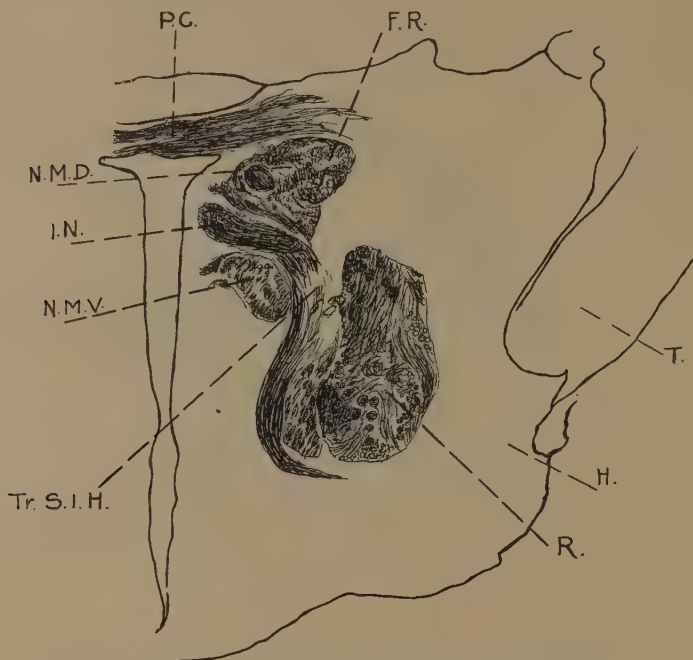


Fig. 3. Transverse section of the brain of *Gasterosteus aculeatus* (silver preparation).

*F.R.* = fasciculus retroflexus; *H.* = hypothalamus; *I.N.* = inner segment of the dorsal thalamus; *N.M.D.* = Nucleus mesencephali dorsalis; *N.M.V.* = Nucleus mesencephali ventralis; *P.C.* = posterior commissure; *T.* = tectum; *Tr. S.I.H.* = Tractus segmento-interno-hypothalamicus.

In the bird, the inner segment of the dorsal thalamus is a region of large size and considerable complexity, comprising several very distinct nuclei. Its position is somewhat more ventral and more lateral than in more primitive classes, on account of the pressure of the cerebellum and cerebral hemispheres, which has pushed all the more dorsal parts of the avian 'tweenbrain and midbrain in a lateral and ventral direction, the optic lobes lying at either side and the other parts being correspondingly shifted in a greater or less degree.

RENDAHL ('24) has recently called attention to the fact that the ventral boundary of the dorsal thalamus (inner segment) is marked medially by a shallow ventricular groove, corresponding to that in amphibians, and although the writers have been unable to detect this sulcus in any of a considerable series of adult avian brains (including *Gallus*, which RENDAHL

figures — loc. cit., fig. 72), they have seen it clearly in chick embryos of about 12 days.

The most antero-dorsal of the nuclei is the anterior nucleus, which is characterized by the presence of a hypothalamic connection, the tractus mamillo-thalamicus (CRAIGIE '28), and has also a large efferent system to the forebrain, the tractus thalamo-striatalis medius. Behind and ventral to the anterior nucleus lies the dorsal nucleus, which also contributes to the medial thalamo-striatal tract; and ventrolateral to this, forming a continuous zone with it — though very distinctly delimited — is the nucleus rotundus. This topographic relation of the nucleus rotundus to the gray matter dorso-medial to it (undifferentiated nuclei anterior and dorsalis) is similar in crocodiles (cf. KAPPERS '21, fig. 461—462). The nucleus rotundus is one of the most conspicuous objects in this region of the avian brain (fig. 4 and 5). It receives fibres from the tectum in the form of a

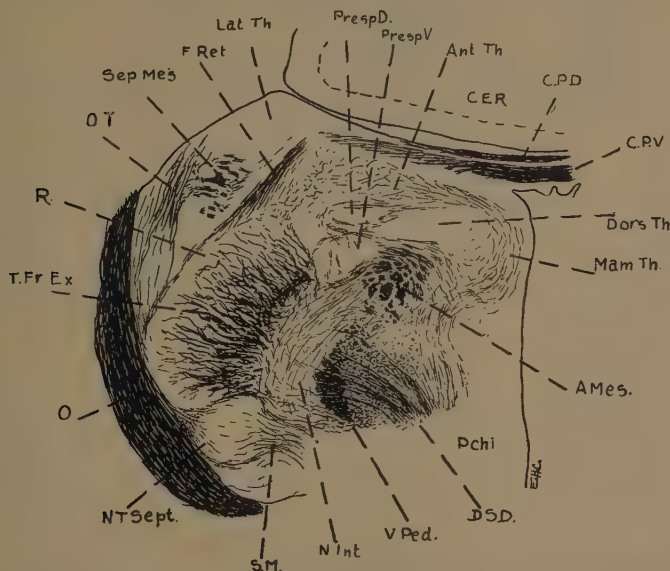


Fig. 4. Part of a transverse section of the brain of humming bird (*Chrysolampis mosquitos* Linn.). The section slants so that ventrally it passes through the preoptic region. (Weigert-Pal preparation).

A. Mes. = archistriato-(occipito-)mesencephalic tract + strio-bulbar tract; Ant Thal. = Nucleus anterior thalami; Cer. = Cerebellum; C.P.D. = Posterior commissure, dorsal part; C.P.V. = Posterior commissure, ventral part; D.S.D. = Dorsal supraoptic decussation; Dors. Th. = Nucleus dorsalis thalami; F. Ret. = Fasciculus retroflexus; Lat. Th. = Nucleus lateralis thalami; Mam Th. = mamillo-thalamic tract; N. Int. = nucleus intercalatus; N. T. Sept. = nucleus tracti septi; O. = optic tract; P. chi = nucleus praearchiasmaticus; Pres. D. = Dorsal praespiriform nucleus; Pres. V. = ventral praespiriform nucleus; R. = nucleus rotundus; Sept. Mes. = septo-mesencephalic tract; T. Fr. Ex. = external thalamo-frontal tract; V. Ped. = ventral forebrain peduncle.



very large tract (tractus tecto-thalamicus) running forward from the posterior region of the latter into its posterior end. It also receives vestibular

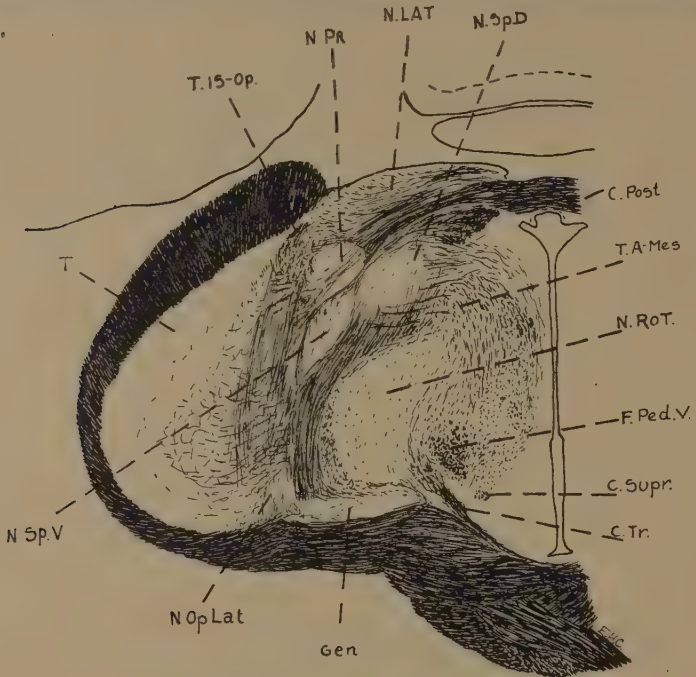


Fig. 5. Transverse section of the brain of a stork (*Ciconia alba*).  
(Weigert-Pal preparation).

*C. Post.* = Posterior commissure; *C. Supr.* = Fibres from the suprainfundibular commissure; *C. Tr.* = Transverse commissure; *F. Ped. V.* = Ventral forebrain peduncle; *Gen.* = Corpus geniculatum (laterale); *N. Lat.* = Lateral nucleus of the thalamus; *N. Op. Lat.* = Lateral optic nucleus; *N. Pr.* = Pretectal nucleus; *N. Rot.* = nucleus rotundus (posterior part. The transversely-cut fibres in the lateral and ventral parts belong to the tecto-thalamic tract); *N. Sp. D.* = Dorso-medial spiriform nucleus; *N. Sp. V.* = Ventro-lateral spiriform nucleus; *T.* = Tectum; *T. A.-Mes.* = Archistriato-mesencephalic tract; *T. Is. Op.* = Isthmo-optic tract.

fibres through the bulbo-thalamic tract (WALLENBERG '04). It has, besides, connections with the spiriform nuclei. From the nucleus rotundus arises the very large lateral thalamo-striatal tract. A smaller nucleus in a more postero-medial situation, the nucleus ovoidalis (CRAIGIE, = nucleus anterior of EDINGER and WALLENBERG), also contributes to the tractus thalamo-striatalis medius.

The small fasciculus retroflexus bears a relationship to the inner segment essentially similar to that in teleosts, amphibians, and reptiles, running postero-ventrally from the habenula to the dorsal surface of the nucleus rotundus, though beyond this point it is difficult to trace it.

The teleostean inner segment, because of its position and connections, has been regarded as homologous with that of amphibia and reptiles (KAPPERS '21; BRICKNER '28).

In comparing this area in teleosts and birds, we must, of course, allow for the displacement of the avian parts, referred to above.

Nevertheless, the general position of the inner segment is very similar in the two groups. The transverse levels of the posterior commissure always include these; the frontal extent is always very great (as far as, or further than the habenula); the sulcus medius, well defined in adult fishes, is present in the corresponding place in chick embryos and the fasciculus retroflexus lies against the area. It is worth mentioning that the same general relations obtain in both amphibian and reptilian brains (cf. for example KAPPERS, '21, figs. 460—464). However, the fasciculus retroflexus of the fish runs immediately caudal to the posterior extremity of the inner segment, while in reptiles and birds it lies not only posterior to the inner segment, but also so far lateral as to form part of the boundary between the inner and outer segments. This displacement is, apparently, due to the growth, particularly caudally, of the inner segment (cf. for example figs. 2 and 3 and DE LANGE '13, fig. 24<sup>1</sup>)).

The connections in fishes and birds, are also very similar. Considering all the factors together, the writers believe the teleostean inner segment to be the relatively small and undifferentiated forerunner of that of the birds. It is, of course, impossible to tell what parts have given rise to particular nuclei, or even whether distinct primordia of these nuclei are present in fishes, but it seems at least that the nucleus rotundus and probably the anterior nucleus are already represented, whether in a somewhat differentiated or in diffuse form. Both of these have large forebrain connections in the bird, as has the segment, very probably, in the fish. The avian nucleus rotundus receives a greatly developed tract from the tectum and this is also seen in the case of the inner segment of the fish, while the anterior nucleus is distinguished by the presence of a mamillo-thalamic tract connecting it with the hypothalamus (fig. 3). It has been seen above, that such a connection is also to be recognized in the fish <sup>2</sup>).

In the fish, a large body, the nucleus mesencephali dorsalis, lies immediately dorsal, and to some extent caudal to the inner segment. While its cells are somewhat diffusely scattered, they are gathered into two main masses, a ventral, which occupies the ventral border of the complex, and a lateral, which lies along the caudo-lateral side of it and has a smaller medial extension. Lateral to the nucleus is the pars anterior of the corpus glomerulosum (fig. 1; nucleus anterior of other authors). These two bodies comprise an extensive region, which we believe to be analogous to

1) Reproduced by KAPPERS ('21) as his fig. 463.

2) It must be pointed out that the tract referred to is not the same as the so-called "Tract. of Vicq d'Azyr" of NILS HOLMGREN. (—20, p. 274—275).

that of the avian *nucleus spiriformis*. This conception depends upon both the positions and the connections of the respective parts.

The nucleus mesencephali dorsalis lies immediately dorsal and the pars anterior of the corpus glomerulosum lateral to the inner segment, in the fish. Correspondingly, the dorso-medial part of the spiriform nucleus is situated exactly dorsal and its ventro-lateral position lateral to the posterior part of the nucleus rotundus, in the bird.

The principal fibre connections of the teleostean nucl. mesencephali dorsalis are with the tectum, the geniculatum, the nucl. intermedius, the posterior commissure, the cerebellum and possibly the torus semicircularis, with a probably descending pathway to the bulb. It has also a connection with the pars anterior of the corpus glomerulosum. The latter structure is linked chiefly with the tectum and the geniculate body dorsally, and through the pars rotundus of the corpus glomerulosum (nucleus rotundus of most authors) with the hypothalamus and the forebrain, ventrally.

The relations in the bird present a remarkable parallel to these. The dorso-medial spiriform nucleus receives fibres from the corpus geniculatum and from the posterior commissure. Some also probably end in it from the archistriato-mesencephalic tract. It gives rise to a descending tract running in the medial longitudinal bundle. The ventro-lateral part has a strong tectal connection, and also receives fibres from the geniculate body and the torus semicircularis. It is furthermore the terminus of the strio-mesencephalic tract.

Thus it seems clear that a close analogy exists between these general regions in fishes and birds, in both of which they receive visual impulses have a forebrain connection, are intimately related to the posterior commissure, and give rise to a descending pathway to the bulb. The vestibular connection by means of the torus semicircularis occurring in the bird is perhaps also paralleled in the fish. The analogy might even be extended to the individual portions of the region, the teleostean nucleus mesencephali dorsalis corresponding with the avian dorso-medial spiriform nucleus and the pars anterior of the corpus glomerulosum with the ventro-lateral spiriform nucleus. The topographic relations of these nuclei in the fish bear almost exactly the respective relations to the inner segment and to each other that the avian structures mentioned bear to the nucleus rotundus and to each other (cf. figs. 1 and 5). In both the nucleus mesencephali dorsalis and the dorso-medial spiriform nucleus we find connections with the geniculatum and the posterior commissure; in both the pars anterior of the corpus glomerulosum and the ventro-lateral spiriform nucleus, we find connections with the tectum, the geniculate body, and the forebrain.

No fibres from the cerebellum have been detected here in the bird, nor has any tract running to the hypothalamus been described. There is, furthermore, a probability that forebrain fibres end in the dorso-medial spiriform nucleus, which does not occur in the nucleus mesencephali dorsalis, and the pars anterior of the corpus glomerulosum has no clear relation to the

posterior commissure. On the other hand, the definite connection between the two nuclei of the fish under discussion is a point in favour of their resemblance to the avian nuclei.

Thus, the parallelism is by no means absolute, but it is sufficient to be distinctly striking, and the discrepancies are perhaps no greater than is to be expected in two groups so widely separated phylogenetically though brought together by convergent evolution.

The lateral cell group of the nucleus mesencephali dorsalis lies in close proximity to the pars anterior of the corpus glomerulosum. It has definite connections with the nucleus intermedius (presumably a part of the geniculatum) and with the posterior commissure. Probably it also shares in all the connections of the nucleus mesencephali dorsalis, and especially with those from the tectum.

The avian *nucleus pretectalis* has a position a little more lateral, possibly due to neurobiotactic attraction toward, or traction by, the displaced tectum; also it lies somewhat more anterior, but it is otherwise very similar to that of the group of cells just described in the fish. It appears to have functional relations similar to those of the spiriform nucleus, its chief fibre-connections being with the posterior commissure and with the tectum. It lies closer to the ventrolateral spiriform nucleus (which has been suggested as the analogue of the pars anterior of the corpus glomerulosum) than to the dorso-medial one and seems to be linked with it by a fibre system. Thus it appears possible that the lateral cell group of the nucleus mesencephali dorsalis of the fish may correspond with the pretectal nucleus of the bird. On the other hand, no part in the brain of the latter can at present be seen to correspond with the so-called pretectal or cortical (FRITSCH) nucleus of the former.

The avian outer segment is likewise more highly differentiated than that of the fish, though less so than the inner segment. It comprises four distinct nuclei, the (lateral) geniculate body, the nucleus intercalatus, the nucleus of the septo-mesencephalic tract, and the lateral nucleus of the thalamus. Its only definitely known components, in the fish, are the geniculate body, nucleus intermedius, and the nucleus subcorticalis (a body lying between the geniculatum and the nucleus intermedius, described in the paper on the teleostean thalamus and midbrain). The latter two should, perhaps, be considered as parts of the geniculatum.

Regarding the ventral thalamus, we can make no specific comparisons. An entopeduncular nucleus is, of course, well known in both classes. In the fish, the only other differentiated nuclei known are the pars rotundus of the corpus glomerulosum and the nucleus mesencephali ventralis, while no the corpus glomerulosum and the nucleus mesencephali centralis, while no other distinct body has been described for the bird except the basal optic ganglion. We see no ground for surprise, however, in the apparent failure of the analogy to extend to this region.

The probable non-existence of an avian part corresponding to the nucleus mesencephali ventralis in the fish is not remarkable, when one considers that a great part of its function is olfactory.

In concluding, the writers have pleasure in expressing their gratitude to Dr. C. U. ARIËNS KAPPERS in whose laboratory it was their privilege to prosecute the investigations out of which the comparisons recorded in the present report have arisen.

### SUMMARY.

1. As a result of independent studies of the brains of birds and of fishes by the respective writers, structural analogies in the midbrain and thalamus of these groups have been observed. This resemblance is not surprising, as these classes are both largely visual in habit and both move in a labile medium, usually without solid support.

2. A homology has been drawn between the inner segment of the dorsal thalamus of the teleost and of the bird, respectively.

3. The avian nucleus rotundus and anterior nucleus are probably represented in undifferentiated form in the teleostean inner segment of the dorsal thalamus.

4. An analogy has been pointed out between the teleostean nucleus mesencephali dorsalis and the pars anterior of the corpus glomerulosum<sup>1)</sup> on the one hand, and the avian spiriform nucleus on the other. The teleostean nuclei mentioned correspond in a marked degree even to the respective parts of the avian cell group. It is possible that a representative of the pretectal nucleus of birds also may be recognized in the lateral cell group of the nucleus mesencephali dorsalis of fishes.

5. The analogy does not appear to extend to structures in the ventral thalamus, except with regard to the entopeduncular nucleus.

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<sup>1)</sup> Nucleus anterior of most authors describing the teleostean brain.



**Mathematics.** — *Zusammenhangsstufen und Cantorsche Mannigfaltigkeiten.* By Dr. K. MENDER. (Communicated by Prof. L. E. J. BROUWER).

(Communicated at the meeting of September 24, 1927).

In früheren Arbeiten habe ich zu wiederholten Malen <sup>1)</sup> darauf hingewiesen, dass, während unsere anschauliche Vorstellung des *Zusammenhanges* nach allen Richtungen durchforscht worden ist, die anschaulich nicht minder klare Vorstellung des *mehr oder minder festen* Zusammenhanges bisher gänzlich ununtersucht geblieben ist. Was uns bei den Worten "Grad des Zusammenhanges" oder "Zusammenhangsstufe" vorschwebt, wird am besten an folgenden Beispielen <sup>2)</sup> deutlich: Eine Quadratfläche ist stärker zusammenhängend als eine Strecke, dagegen schwächer zusammenhängend als ein Würfel. Eine Menge, welche Summe ist von zwei Quadratflächen, die nur einen Eckpunkt mit einander gemein haben, ist schwächer zusammenhängend als eine Quadratfläche. Ich habe in früheren Arbeiten auch betont <sup>3)</sup>, dass der Begriff der verschiedenen Zusammenhangsstufen nicht nur an sich von einem gewissen Interesse ist, sondern von grosser Wichtigkeit werden dürfte für die Topologie des  $R_n$  für die topologische Charakterisierung des  $n$ -dimensionalen Simplexes und für andere topologische Fragen. Ich habe dabei insbesondere auf die Begriffsbildungen des  $n$ -stufigen *Kontinuums*, des  $n$ -stufig *irreduzibeln* Kontinuums, der  $n$ -stufig irreduzibel zusammenhängenden Mengen und der *im kleinen  $n$ -stufig zusammenhängenden Mengen* hingewiesen <sup>4)</sup>, — Begriffsbildungen, deren Behandlung ein weites Arbeitsgebiet eröffnet.

Eine strenge Präzisierung <sup>5)</sup> der Vorstellung von den verschiedenen Zusammenhangsstufen erfolgt am einfachsten durch folgende Definition: *Ein Raum heisst (mindestens)  $n$ -stufig zusammenhängend, wenn er nicht*

<sup>1)</sup> Vgl.: a. Ueber die Dimension von Punktmengen I. Monatshefte f. Math. u. Phys. Bd. 33, S. 160.

b. Ueber die Dimension von Punktmengen II. Monatshefte Bd. 34, S. 161.

c. Bericht über die Dimensionstheorie, Jahresber. d. deutsche Math. Ver. Bd. 35, S. 144 ff.

<sup>2)</sup> l.c. <sup>1)</sup> c. S. 144.

<sup>3)</sup> l.c. <sup>1)</sup> a. S. 161; b. S. 156.

<sup>4)</sup> l.c. <sup>1)</sup> c. S. 145. Ein Raum soll *im kleinen  $n$ -stufig zusammenhängend* heissen, wenn zu jedem Punkte beliebig kleine  $n$ -stufig zusammenhängende Umgebungen existieren. M.a.W. Wenn das System der Umgebungen des Raumes mit einem System von  $n$ -stufig zusammenhängenden Umgebungen gleichwertig ist. Oder, was in einem metrischen Raum auf dasselbe hinausläuft, wenn der Raum für jedes  $\varepsilon > 0$  Summe ist von endlich vielen  $n$ -stufigen Kontinua mit Durchmessern  $< \varepsilon$ .

<sup>5)</sup> l.c. <sup>1)</sup> c. S. 144 f.

Summe ist von zwei echten<sup>6)</sup> abgeschlossenen Teilmengen mit  $(n-1)$ -stufig zusammenhangslosem Durchschnitt. Dabei heisst  $(n-1)$ -stufig zusammenhangslos ein Raum, welcher keinen  $(n-1)$ -stufig zusammenhängenden Teil enthält. Die leere Menge und nur diese ist nullstufig zusammenhangslos.

Durch die Festsetzung der leeren Menge als Ausgangspunkt der Rekursion ergeben sich als (mindestens) einstufig zusammenhängend die im gewöhnlichen (Lennes-Hausdorffschen) Sinn zusammenhängenden Räume. Die Strecke und ebenso die Summe von zwei Quadratlflächen, die bloss einen Eckpunkt mit einander gemein haben, ist einstufig, aber nicht zweistufig zusammenhängend. Die letztere Menge enthält jedoch zweistufig zusammenhängende Teilmengen. Die Quadratfläche ergibt sich nämlich als zweistufig zusammenhängend; der Würfel als dreistufig zusammenhängend, u.s.w.

Bei der Aufstellung dieser Begriffsbildung ergab sich mir die Frage<sup>7)</sup>, nach den Beziehungen von *Zusammenhangsstufe* und *Dimension* von Räumen. Man sieht durch einen einfachen Induktionsschluss: Jeder Raum, welcher eine mindestens  $n$ -stufig zusammenhängende Teilmenge enthält, ist mindestens  $n$ -dimensional ( $n \geq 1$ ), oder, was gleichbedeutend ist: Ein höchstens  $n$ -dimensionaler Raum ist  $(n+1)$  stufig zusammenhangslos. Für  $n=0$  ist diese Aussage trivial. Wir nehmen an, es sei bereits bewiesen, dass ein höchstens  $(n-1)$ -dimensionaler Raum  $n$ -stufig zusammenhangslos ist. Es sei dann  $R$  irgend ein gegebener  $n$ -dimensionaler Raum ( $n \geq 1$ ). Wir machen die Annahme,  $R$  enthalte eine mindestens  $(n+1)$ -stufig zusammenhängende Teilmenge  $M$ , und leiten aus dieser Annahme einen Widerspruch her. Da  $R$  mehr als nulldimensional ist, enthält  $R$  sicherlich zwei verschiedene Punkte  $p$  und  $q$ . Da  $R$  höchstens  $n$ -dimensional ist, existiert eine Umgebung  $U$  von  $p$  mit höchstens  $(n-1)$ -dimensionaler Begrenzung, die so klein ist, dass  $q$  im Komplement der abgeschlossenen Hülle  $\bar{U}$  von  $U$  liegt. Durch die Formel

$$M = M \cdot \bar{U} + M \cdot (R - U)$$

wird  $M$  dargestellt als Summe zweier in  $M$  abgeschlossener Mengen, deren Durchschnitt (d.i. die Begrenzung von  $U$ ) höchstens  $(n-1)$ -dimensional, also nach Annahme  $n$ -stufig zusammenhangslos ist und wobei jeder der beiden Summanden eine echte Teilmenge von  $M$  ist, denn der eine enthält  $p$ , aber nicht  $q$  und der andere enthält  $q$ , aber nicht  $p$ . Da dies der Annahme,  $M$  sei  $(n+1)$ -stufig zusammenhängend, widerspricht, ist diese Annahme ad absurdum geführt und unsere Behauptung bewiesen.

Die umgekehrte Behauptung, der Satz nämlich, dass jeder  $(n+1)$ -stufig

<sup>6)</sup> Die Beschränkung auf Zerlegungen in *echte* abgeschlossene Teilmengen ist wesentlich. Denn würde man auch Zerlegungen zulassen, bei denen einer der Summanden mit dem ganzen Raum identisch und der andere etwa leer ist, so wäre ja jeder Raum, Summe von zwei abgeschlossenen Teilmengen, deren Durchschnitt leer ist, also unzusammenhängend.

l.c. <sup>1)</sup>

zusammenhangslose Raum höchstens  $n$ -dimensional ist, ist bekanntlich unrichtig, da SIERPIŃSKI<sup>8)</sup> eine zusammenhangslose (also in unserer Terminologie einstufig zusammenhangslose) eindimensionale Menge konstruiert hat. Hingegen gilt bekanntlich<sup>9)</sup>, der Satz, dass jeder kompakte oder halbkompakte (einstufig) zusammenhangslose Raum nulldimensional ist. Ich stellte mir daher die Frage<sup>10)</sup>: Sind unter den kompakten und halbkompakten Räumen die  $n$ -dimensionalen durch  $(n+1)$ -stufige Zusammenhangslosigkeit charakterisiert?

Ich will nun im Folgenden einige Konsequenzen angeben, welche eine positive Antwort auf diese Frage hat, u.zw. Konsequenzen, welche sich auf die Cantorsche Mannigfaltigkeiten im Sinne von URYSOHN beziehen und durch welche dieser Begriff in meine Begriffsbildung des  $n$ -stufigen Zusammenhanges eingeordnet wird. URYSOHN bezeichnet<sup>11)</sup> als  $n$ -dimensionale Cantorsche Mannigfaltigkeit einen  $n$ -dimensionalen kompakten Raum, der nach Tilgung einer beliebigen  $(n-2)$ -dimensionalen Teilmenge zusammenhängend bleibt. URYSOHN hat das folgende Problem gestellt (und dessen besondere Wichtigkeit und Schwierigkeit betont<sup>12)</sup>: Enthält jeder kompakte  $n$ -dimensionale Raum eine  $n$ -dimensionale Cantorsche Mannigfaltigkeit als Teilmenge? Ich will nun zunächst zeigen, dass diese Frage äquivalent ist mit meinem oben erwähnten Problem, ob unter den kompakten Räumen die  $n$ -dimensionalen durch  $(n+1)$ -stufige Zusammenhangslosigkeit gekennzeichnet sind.

Machen wir nämlich erstens die Annahme I: Jeder  $n$ -dimensionale kompakte Raum enthält eine  $n$ -stufig zusammenhängende Teilmenge. Wir zeigen, es folgt daraus die Behauptung II: Jeder  $n$ -dimensionale kompakte Raum enthält eine  $n$ -dimensionale Cantorsche Mannigfaltigkeit als Teilmenge. Mit Rücksicht auf die Annahme I genügt es zum Beweis von II, aus der Annahme I folgende Behauptung herzuleiten: Jeder  $n$ -dimensionale,  $n$ -stufig zusammenhängende kompakte Raum ist eine  $n$ -dimensionale Cantorsche Mannigfaltigkeit. Angenommen nun,  $R$  sei ein  $n$ -dimensionaler kompakter Raum, welcher keine  $n$ -dimensionale Cantorsche Mannigfaltigkeit ist, welcher also eine höchstens  $(n-2)$ -dimensionale abgeschlossene Teilmenge  $A$  enthält, deren Komplement nicht zusammenhängend, also Summe von zwei fremden abgeschlossenen nicht-leeren Teilmengen  $R'$  und  $R''$  ist. Durch die Formel  $R = (R' + A) + (R'' + A)$  ist  $R$  dargestellt als Summe zweier abgeschlossener echter Teilmengen, deren Durchschnitt die Menge  $A$  ist, deren Durchschnitt also höchstens  $(n-2)$ -dimensional und mithin nach Annahme I  $(n-1)$ -stufig zusammenhangslos ist. Also ist  $R$  höchstens  $(n-1)$ -stufig zusammenhängend. Ein  $n$ -stufig zusammenhängender  $n$ -dimensionaler kompakter Raum ist also,

<sup>8)</sup> Fund. Math. 2, S. 81.

<sup>9)</sup> Vgl. etwa l.c. 1) a. S. 159.

<sup>10)</sup> l.c. 1) b. S. 156.

<sup>11)</sup> Fund. Math. 7, S. 124.

<sup>12)</sup> Fund. Math. 8, S. 285.

wenn Annahme I gilt, eine  $n$ -dimensionale Cantorsche Mannigfaltigkeit, womit gezeigt ist, dass aus der Annahme I die Behauptung II folgt.

Machen wir nun *zweitens* die Annahme, es gelte die Behauptung II. Wir wollen aus ihr die Behauptung I herleiten. Diese Behauptung I ist, wie oben erwähnt, sicher für  $n=1$  richtig. Wir nehmen an, es sei bewiesen, dass jede  $(n-1)$ -dimensionale Cantorsche Mannigfaltigkeit  $(n-1)$ -stufig zusammenhängend ist, was für  $n=1$  gleichfalls richtig ist. Aus dieser Annahme folgt auf Grund von Annahme II: Jeder  $(n-1)$ -stufig zusammenhangslose kompakte Raum ist höchstens  $(n-2)$ -dimensional. Sei nun  $R$  ein  $n$ -dimensionaler kompakter Raum, welcher nicht  $n$ -stufig zusammenhängend ist, welcher also Summe ist von zwei abgeschlossenen echten Teilmengen  $R'$  und  $R''$ , deren Durchschnitt  $(n-1)$ -stufig zusammenhangslos ist. Auf Grund der Annahme II ist der Durchschnitt von  $R'$  und  $R''$  höchstens  $(n-2)$ -dimensional, also ist  $R$  keine Cantorsche Mannigfaltigkeit, m.a.W.: Aus der Annahme II folgt, dass jede  $n$ -dimensionale Cantorsche Mannigfaltigkeit  $n$ -stufig zusammenhängend ist; also folgt aus der Behauptung II die Behauptung I. Damit ist also gezeigt:

*Die beiden folgenden Sätze sind mit einander äquivalent:*

I. *Jeder  $n$ -dimensionale kompakte Raum enthält einen  $n$ -stufig zusammenhängenden Teil.*

II. *Jeder kompakte  $n$ -dimensionale Raum enthält eine  $n$ -dimensionale Cantorsche Mannigfaltigkeit als Teilmenge.*

Da, wie oben bewiesen, die höchstens  $n$ -dimensionalen Räume  $(n+1)$ -stufig zusammenhangslos sind, so ist mit den beiden angeführten Sätzen auch der folgende äquivalent:

III. *Unter den kompakten Räumen sind die höchstens  $n$ -dimensionalen und die  $(n+1)$ -stufig zusammenhangslosen identisch.*

Aus dem Beweise der Äquivalenz der Sätze I und II geht überdies hervor, dass, falls die drei angeführten Sätze richtig sind, auch folgender Satz gilt, welcher den Begriff der Cantorschen Mannigfaltigkeit in die Begriffsbildung des höherstufigen Zusammenhanges einordnet:

IV. *Die Cantorschen Mannigfaltigkeiten sind jene kompakten Räume, für welche Dimension und Zusammenhangsstufe übereinstimmen. Die  $n$ -dimensionalen Cantorschen Mannigfaltigkeiten sind identisch mit den  $n$ -dimensionalen  $n$ -stufig zusammenhängenden kompakten Räumen.*

Dass die vier im Vorangehenden als äquivalent erwiesenen Sätze tatsächlich gelten, wurde gemeinsam von HUREWICZ und mir bewiesen<sup>13)</sup>. Der Gedankengang unseres Beweises ist folgender:

Bekanntlich ist jeder kompakte  $n$ -dimensionale Raum für jedes  $\varepsilon > 0$  Summe von endlich vielen abgeschlossenen Mengen mit Durchmessern  $< \varepsilon$ , die zu je  $n+2$  fremd sind<sup>14)</sup>, aber nicht für jedes  $\varepsilon > 0$  Summe

<sup>13)</sup> Vgl. unsere ausführliche Darstellung, welche demnächst in den Mathem. Annalen unter dem Titel "Zusammenhangsstufen und Dimension" erscheint.

<sup>14)</sup> Vgl. l.c. 1) b. S. 153 und URYSOHN, Fund. Math. 8, S. 292.

von endlich vielen abgeschlossenen Mengen  $< \varepsilon$ , die zu je  $n+1$  fremd sind <sup>15)</sup>. Dies rechtfertigt folgende Bezeichnungsweise: Ein kompakter Raum heie *n-dimensional von Grade  $r$* , wenn  $r$  die kleinste reelle Zahl ist von der Art, dass es zu jeder reellen Zahl  $r' > r$  eine Zerlegung von  $R$  gibt in endlich viele abgeschlossene Teilmengen mit Durchmessern  $< r'$ , die zu je  $n+1$  fremd sind <sup>16)</sup>. Es ist in dieser Ausdrucksweise ein kompakter Raum *n-dimensional* dann und nur dann, wenn er vom Grade Null  $(n+1)$ -dimensional, und von irgend einem Grade  $r > 0$  *n-dimensional* ist.

Man sieht nun mheles ein, dass fr jede reelle Zahl  $r$  die Eigenschaft eines kompakten Raumes *n-dimensional von einem Grade  $\equiv r$*  zu sein, eine induzible Eigenschaft ist, d.h.: Ist  $n$  eine natrliche Zahl und  $r$  eine reelle Zahl  $> 0$  und ist  $\{A_k\}$  ( $k = 1, 2, \dots$  ad inf.) eine monoton abnehmende Folge von abgeschlossenen Teilmengen eines kompakten Raumes, von denen jede von einem Grade  $\equiv r$  *n-dimensional* ist, dann ist auch der Durchschnitt  $A = \bigcap_{k=1}^{\infty} A_k$  von einem Grade  $\equiv r$  *n-dimensional*.

Nach dem BROUWERSchen Reduktionstheorem enthlt daher jeder kompakte *n-dimensionale* Raum  $R$  eine abgeschlossene Teilmenge  $M$ , welche von demselben Grad *n-dimensional* ist, wie  $R$  selbst, welche aber hinsichtlich dieser Eigenschaft irreduzibel ist, d.h. so, dass keine echte Teilmenge von  $M$  von demselben Grade wie  $R$  *n-dimensional* ist. Diese Teilmenge  $M$  von  $R$  ist aber eine *n-dimensionale Cantorsche Mannigfaltigkeit*. Denn nach HUREWICZ <sup>17)</sup> gilt folgender *Additionssatz*: Sind  $A$  und  $A'$  zwei abgeschlossene Teilmengen eines kompakten Raumes, deren jede *n-dimensional von einem Grade  $\equiv r$*  ist, und deren Durchschnitt hchstens  $(n-2)$ -dimensional ist, dann ist auch die Menge  $A + A'$  *n-dimensional von einem Grade  $\equiv r$* . Aus diesem Satz folgt aber unmittelbar, dass jeder *n-dimensionale* kompakte Raum, welcher keine *n-dimensionale Cantorsche Mannigfaltigkeit* ist, eine echte abgeschlossene Teilmenge enthlt, welche von demselben Grade *n-dimensional* ist, wie der Gesamtraum. Daher ist in einem *n-dimensionalen* kompakten Raum  $R$  die Teilmenge  $M$ , (deren Existenz wir oben nachgewiesen haben), welche von demselben Grade *n-dimensional* ist wie  $R$ , aber keinen echten Teil enthlt, welcher von demselben Grade *n-dimensional* wie  $R$  ist, eine *n-dimensionale Cantorsche Mannigfaltigkeit*.

<sup>15)</sup> Vgl. URYSOHN, Fund. Math. 8, S. 294.

<sup>16)</sup> Vgl. hierzu URYSOHN, Fund. Math. 8, S. 353.

<sup>17)</sup> Vgl. Proc. fAc. Amsterdam, 29, 1926 (Stetige Bilder, II) und den ausfhrlichen Beweis, den HUREWICZ in einem Anhang zu unserer sub <sup>13)</sup> erwhnten gemeinsamen Arbeit von dem Additionssatz gibt.



**Mathematics.** — *Bemerkungen zur zweiten Untersuchung über allgemeine Metrik.* By Dr. K. Menger. (Communicated by Prof. L. E. J. Brouwer).

(Communicated at the meeting of September 24, 1927).

Eine Analyse der Ueberlegungen, welche in der zweiten Untersuchung über allgemeine Metrik <sup>1)</sup> zu einer Charakterisierung der euklidischen Metrik geführt haben, fördert einige allgemeine Begriffe und Aussagen über metrische Räume zu Tage, deren Anwendbarkeit über die euklidischen Räume erheblich hinausgeht.

1. DER BEGRIFF DER KONGRUENZZORDNUNG. Unsere Charakterisierung der euklidischen Metrik basiert auf dem Satz <sup>2)</sup>: Ein halbmétrischer Raum <sup>3)</sup>, von dem je  $n+3$  Punkte in den  $R_n$  (den  $n$ -dimensionalen euklidischen Raum) abstandstreu einbettbar sind, ist selbst in den  $R_n$  abstandstreu einbettbar. Diese Tatsache drückt eine Eigenschaft  $R_n$  aus, welche auch manchen anderen metrischen Räumen zukommt. Wir sagen: *Ein halbmétrischer Raum  $A$  besitzt die Kongruenzordnung  $k$ , wenn jeder Raum, von dem je  $k$  Punkte in  $A$  abstandstreu einbettbar sind, in  $A$  abstandstreu einbettbar ist.* Offenbar ist jeder Raum, der die Kongruenzordnung  $k$  besitzt, zugleich auch von jeder Kongruenzordnung  $m > k$ . Der  $R_n$  ist in dieser Ausdrucksweise von der Kongruenzordnung  $n+3$ . Der Halb- $R_n$  hingegen ist von keiner endlichen Kongruenzordnung. Denn es ist ja beispielsweise jede endliche (ja sogar jede beschränkte) Teilmenge des  $R_n$  in den Halb- $R_n$  abstandstreu einbettbar, ohne dass der  $R_n$  selbst in den Halb- $R_n$  abstandstreu einbettbar wäre. Der  $R_n$  enthält auch Teilmengen, die eine endliche Kongruenzordnung besitzen, aber nur eine grössere als der  $R_n$  selbst. Beispielsweise ist der  $R_1$  von der Kongruenzordnung 4. Betrachten wir aber die Teilmenge  $A$  des  $R_1$  bestehend aus den Punkten des  $R_1$  mit den Koordinaten 0,1,2,3,5,6, so besitzt sie, als Raum aufgefasst, nicht die Kongruenzordnung 4; denn betrachten wir die Menge  $B$  des  $R_1$  bestehend aus den Punkten mit den Koordinaten 0,1,2,3,4, — so sehen wir, dass je vier Punkte von  $B$  in  $A$  abstandstreu einbettbar sind, ohne dass  $B$  in  $A$  abstandstreu einbettbar wäre. Die Menge  $A$  besitzt aber doch eine

<sup>1)</sup> Mathematische Annalen Bd. 98 oder 99 (Im Folgenden zitiert als "M. II").

<sup>2)</sup> M. II.

<sup>3)</sup> Als halbmétrischen Raum bezeichnen wir (vgl. M. II) eine Menge, in der je zwei Elementen  $p$  und  $q$  eine Zahl  $pq = qp > 0$  für  $p \neq q$  und  $pq = 0$  für  $p = q$  zugeordnet ist.

endliche Kongruenzordnung, den offenbar ist jeder Raum, welcher  $n$  Punkte enthält, von einer Kongruenzordnung  $\leq n + 1$  <sup>4)</sup>).

2. DER BEGRIFF DER METRISCHEN BASIS. Ist  $A$  ein halbmetrischer Raum,  $B$  eine Teilmenge von  $A$ , so sagen wir,  $B$  sei eine *metrische Basis* von  $A$ , wenn es keine zwei verschiedenen Punkte  $p$  und  $q$  von  $A$  gibt, so dass jeder Punkt von  $A$  denselben Abstand von  $p$  wie von  $q$  hat, m.a.W., wenn jeder Punkt von  $A$  durch seinen Abstand von den Punkten der Menge  $B$  eindeutig gekennzeichnet ist. In einem halbmetrischen Raum mit stetiger Metrik ist jede im Raum dichte Teilmenge eine metrische Basis. Jeder separable metrische Raum besitzt also eine abzählbare metrische Basis. Im  $R_n$  ist eine Menge  $B$  dann und nur dann metrische Basis, wenn sie  $n + 1$  nicht in einer Hyperebene des  $R_n$  gelegene Punkte enthält. Je  $n + 1$  Punkte des  $R_n$ , die nicht in einer Hyperebene des  $R_n$  liegen, bilden eine metrische *Minimalbasis* des  $R_n$ , d.h. eine metrische Basis, von der keine echte Teilmenge metrische Basis der  $R_n$  ist. Falls in einem halbmetrischen Raum mit stetiger Metrik eine metrische Minimalbasis existiert, so ist sie stets eine Teilmenge, die keinen ihrer Häufungspunkte (wofern solche existieren) enthält. Es bilden  $k$  Punkte eines metrischen Raumes offenbar dann und nur dann eine metrische Basis desselben, wenn die Begrenzungen von je  $k$  Kugelumgebungen mit den betreffenden Punkten als Zentren höchstens einen Punkt gemein haben.

Eine metrische Basis  $B$  des Raumes  $A$  heisst *vollkommen*, wenn zu jeder Teilmenge  $M'$  von  $A$ , welche einen mit  $B$  kongruenten Teil  $B'$  enthält, eine kongruente Menge  $M$  existiert, die  $B$  enthält. Beispielsweise ist jede Basis eines  $R_n$  vollkommen. Betrachten wir hingegen einen Halb- $R_n$ , oder einen Raum, welcher aus dem  $R_n$  durch Weglassung eines einzigen Punktes entsteht, so sehen wir: diese Räume besitzen Basen; auch in ihnen bilden nämlich je  $n + 1$  Punkte, welche nicht in einer Hyperebene des  $R_n$  liegen, eine Basis (u. zw. eine Minimalbasis); aber diese Räume besitzen offenbar keine vollkommene Basis. Man sieht aus diesen Beispielen, dass die Existenz vollkommener Basen mit einer gewissen metrischen Vollständigkeit der Räume bzw. mit einer metrischen Homogenität, einer kongruenten Selbstabbildbarkeit der Räume zusammenhängt.

3. BEGRIFF DER MENGENSYMENTRALEN. Ist  $A$  eine gegebene Teilmenge eines halbmetrischen Raumes, so bezeichnen wir die (eventuell leere) Menge aller jener Punkte des Raumes, welche von sämtlichen Punkten der Menge  $A$  den gleichen Abstand haben, mit  $S(A)$  und

<sup>4)</sup> Für jede natürliche Zahl  $n$  existieren Räume, welche die Kongruenzordnung  $n + 1$ , aber keine niedrigere Kongruenzordnung besitzen. (Beispielsweise Räume, welche aus  $n$  Punkten bestehen, von denen je zwei den Abstand 1 besitzen). Die Räume, welche aus drei Punkten bestehen, sind sogar durchwegs von keiner niedrigeren Ordnung als vier. Hingegen gibt es Räume, welche aus vier Punkten bestehen und von der Kongruenzordnung drei sind, beispielsweise die antieuklidischen Quadrupel, (vgl. M. II).

nennen diese Menge die *Symmetrale der Menge A*. Besteht die Menge  $A$  aus einem einzigen Punkt, so ist  $S(A)$  mit dem ganzen Raum identisch. Enthält  $A$  mehr als einen Punkt, so ist die Menge  $S(A)$  zu  $A$  fremd, also eine (eventuell leere) echte Teilmenge des Raumes. Jede echte Teilmenge des Raumes, welche Symmetrale irgend einer Menge des Raumes ist, m. a. W. jede Menge  $S$ , zu welcher eine mehr als einen Punkt enthaltene Menge  $A$  existiert, so dass  $S = S(A)$  gilt, nennen wir auch kurz eine *Symmetrale*. Offenbar gilt für je zwei Mengen  $A$  und  $B$  die Beziehung  $S(A + B) = S(A) \cdot S(B)$ . In einem Raum mit stetiger Metrik ist jede Symmetrale eine *abgeschlossene Menge* und gilt ferner für jede Menge  $M$ , wenn  $\bar{M}$  die abgeschlossene Hülle von  $M$  bezeichnet,  $S(\bar{M}) = S(M)$ . In Räumen mit stetiger Metrik kann man sich also auf die Betrachtung der Symmetralen *abgeschlossener Mengen* beschränken.

Es besteht nun folgende einfache Beziehung: *Jede Teilmenge eines metrischen Raumes ist entweder metrische Basis des Raumes oder Teilmenge einer Symmetralen*. In der Tat, wenn *erstens*  $M$  Teilmenge einer Symmetralen ist, so existieren zwei verschiedene Punkte  $p$  und  $q$ , so dass jeder Punkt von  $M$  denselben Abstand von  $p$  wie von  $q$  hat. Dann ist aber  $M$  keine metrische Basis des Raumes. Ist *zweitens*  $M$  keine metrische Basis des Raumes, dann existieren zwei verschiedene Punkte  $p$  und  $q$ , so dass jeder Punkt von  $M$  denselben Abstand von  $p$  wie von  $q$  hat. Dann ist aber  $M$  Teilmenge der Symmetrale der aus Punkte  $p$  und  $q$  bestehenden Menge.

4. EIN ALLGEMEINER SATZ. Der tiefere Grund dafür, dass der  $R_n$  die Kongruenzordnung  $n + 3$  hat, ist nun in folgendem allgemeinem Satz gelegen.

*Voraussetzungen: Es sei A ein halbmétrischer Raum mit folgenden Eigenschaften:*

1. *In A ist ein System von untereinander kongruenten Teilmengen definiert (welche wir kurz als Unterräume von A bezeichnen), die, in sich betrachtet, Räume von der Kongruenzordnung  $n + 2$  sind.*

2. *Liegen von  $n + 2$  gegebenen Punkten je  $n + 1$  in einem Unterraum, so liegen alle  $n + 2$  Punkte in einem und demselben Unterraum.<sup>5)</sup>*

3. *Je  $n + 1$  Punkte von A liegen entweder in einem Unterraum oder sind mit einer vollkommenen metrischen Basis des Raumes kongruent.*

*Behauptung: Der Raum A besitzt die Kongruenzordnung  $n + 3$ .*

BEWEIS: Es sei ein Raum  $R$  gegeben, von dem je  $n + 3$  Punkte abstandstreu in  $A$  einbettbar sind. Wir haben zu zeigen, dass  $R$  in  $A$  abstandstreu einbettbar ist. Nun sind zwei Fälle möglich:

<sup>5)</sup> Man beachte, dass diese Voraussetzung keineswegs besagt, dass die Unterräume die Kongruenzordnung  $n + 1$  besitzen. (Dieselben sind laut Voraussetzung 1) von der Kongruenzordnung  $n + 2$ ). Denn es handelt sich bei der Voraussetzung 2) nicht um abstands-treue Abbildbarkeit, sondern um Enthaltensein.

Entweder sind je  $n+2$  Punkte von  $R$  in einen Unterraum von  $A$  abstandstreu einbettbar. Dann ist  $R$ , da die Unterräume nach Voraussetzung 1) untereinander kongruent und von der Kongruenzordnung  $n+2$  sind, in einen Unterraum, d.i. in eine Teilmenge von  $A$ , also erst recht in  $A$  abstandstreu einbettbar.

Oder es gibt  $n+2$  Punkte  $p_1, p_2, \dots, p_{n+2}$  von  $R$ , die in  $A$ , aber nicht in einen Unterraum von  $A$  abstandstreu einbettbar sind. Wir betrachten ein zu ihnen kongruentes System  $a_1, a_2, \dots, a_{n+2}$  von Punkten aus  $A$ , wobei  $a_i$  dem Punkt  $p_i$  ( $i = 1, 2, \dots, n+2$ ) entsprechen möge. Die Punkte  $a_1, a_2, \dots, a_{n+2}$  liegen nicht in einem Unterraum von  $A$ . Dann gibt es aber unter ihnen nach Voraussetzung 2)  $n+1$  Punkte, die nicht in einem Unterraum liegen, und daher nach Voraussetzung 3) mit einer vollkommenen metrischen Basis von  $A$  kongruent sind. Diese vollkommene Basis möge aus den Punkten  $b_1, b_2, \dots, b_{n+1}$  bestehen, und etwa mit den Punkten  $p_1, p_2, \dots, p_{n+1}$  kongruent sein. Es sei nun  $p$  irgend ein Punkt von  $R$ . Da je  $n+3$  Punkte und daher erst recht je  $n+2$  Punkte von  $R$  in  $A$  abstandstreu einbettbar sind, existiert ein zu den Punkten  $p_1, p_2, \dots, p_{n+1}, p$  kongruentes Punktesystem in  $A$ . Also existiert, da die aus den Punkten  $b_1, b_2, \dots, b_{n+1}$  bestehende Basis nach Voraussetzung vollkommen ist, ein Punkt  $p'$  von  $A$ , so dass das System  $b_1, b_2, \dots, b_{n+1}, p'$  zum System  $p_1, p_2, \dots, p_{n+1}, p$  kongruent ist. Es existiert nur ein einziger Punkt  $p'$  von dieser Eigenschaft, da die Punkte  $b_1, b_2, \dots, b_{n+1}$  eine metrische Basis von  $A$  bilden. Auf diese Weise wird also jedem Punkt  $p$  von  $R$  ein einziger Punkt  $p'$  von  $A$  zugeordnet, d.h. es wird  $R$  eindeutig auf eine Teilmenge von  $A$  abgebildet. Wir behaupten, dass die so definierte Abbildung *abstandstreu* ist. Seien nämlich  $p$  und  $q$  irgend zwei Punkte von  $R$ ,  $p'$  und  $q'$  ihre Bilder vermöge der definierten Abbildung in  $A$ . Da je  $n+3$  Punkte von  $R$  in  $A$  abstandstreu einbettbar sind, existiert ein mit den Punkten  $p_1, p_2, \dots, p_{n+1}, p, q$  kongruentes Punktesystem in  $A$ . Dasselbe enthält ein mit den Punkten  $p_1, p_2, \dots, p_{n+1}$  und daher ein mit den Punkten  $b_1, b_2, \dots, b_{n+1}$  kongruentes Teilsystem. Wegen der Voraussetzung, dass die aus den Punkten  $b_1, b_2, \dots, b_{n+1}$  bestehende metrische Basis vollkommen ist, existiert daher ein zum System der Punkte  $p_1, p_2, \dots, p_{n+1}, p, q$  kongruentes Punktesystem, welches die Punkte  $b_1, b_2, \dots, b_{n+1}$  enthält, etwa das System  $b_1, b_2, \dots, b_{n+1}, p^*, q^*$ . Nun hat der Punkt  $p^*$  von sämtlichen Punkten  $b_1, b_2, \dots, b_{n+1}$  dieselben Abstände wie der Punkt  $p'$ , also sind, da die Punkte  $b_1, b_2, \dots, b_{n+1}$  eine metrische Basis von  $A$  bilden, die Punkte  $p^*$  und  $p'$  identisch. Aus demselben Grunde sind  $q^*$  und  $q'$  identisch. Mithin ist der Abstand von  $p'$  und  $q'$  gleich dem Abstand von  $p^*$  und  $q^*$ , d.i. gleich dem Abstand von  $p$  und  $q$ . Da dies für je zwei Punkte  $p$  und  $q$  von  $R$  gilt, ist gezeigt, dass die oben definierte Abbildung von  $R$  auf eine Teilmenge von  $A$  abstandstreu ist, womit unser Satz bewiesen ist.

Die Bedingungen unseres Satzes, welche dafür hinreichend sind, dass der Raum  $A$  die Kongruenzordnung  $n+3$  besitzt, lassen sich, wie hier

noch bemerkt werden möge, noch abschwächen und in verschiedener Weise durch andere ersetzen. Beispielsweise können auf Grund der obigen Bemerkungen über Symmetralmengen die Bedingungen 1 und 3 ersetzt werden durch die folgenden:

1'. Für je zwei Paare von verschiedenen Punkten von  $A$ , für  $p, q$  und  $p', q'$  sind die Mengen  $S(p, q)$  und  $S(p', q')$  mit einander kongruent. Jede Menge  $S(p, q)$ , wo  $p$  und  $q$  zwei verschiedene Punkte von  $A$  sind, ist, in sich betrachtet, ein Raum von der Kongruenzordnung  $n+2$ .

3'. Jede metrische Basis von  $A$  ist kongruent mit einer vollkommenen Basis von  $A$ .

Aus den Bedingungen 1' und 3' folgen nämlich die Bedingungen 1 und 3. Bezeichnen wir das System der Mengen  $S(p, q)$  für alle Paare von verschiedenen Punkten von  $A$  als das System der Unterräume von  $A$ , so wird, mit Rücksicht auf die Bedingung 1' der Bedingung 1 genügt. Ferner bilden je  $n+1$  Punkte, welche nicht in einem derartigen Unterraum liegen, eine metrische Basis und daher wegen Bedingung 3' eine vollkommene metrische Basis von  $A$ , so dass Bedingung 3 erfüllt ist.

5. ANWENDUNGEN. Von den Anwendungen des bewiesenen Satzes, seien hier nur die folgenden erwähnt. Der  $R_0$ , der nulldimensionale euklidische Raum, d.h. ein aus genau einem Punkt bestehender Raum, ist von der Kongruenzordnung 3 (sogar von der Kongruenzordnung 2). Durch Induktion verifiziert man mühelos, dass der  $R_n$  die Bedingungen 1, 2, 3 des allgemeinen Satzes erfüllt, wofern als Unterräume die Hyper-ebenen des  $R_n$  gewählt werden. Dieselben Ueberlegungen sind für die  $n$ -dimensionalen nicht-euklidischen Räume giltig.

Die nulldimensionale Sphäre, d.i. ein aus genau zwei Punkten bestehender Raum, besitzt die Kongruenzordnung 3. Durch Induktion verifiziert man ebenfalls ohne Schwierigkeiten, dass die  $n$ -dimensionale Sphäre die Bedingungen 1, 2, 3 unseres Satzes erfüllt, wofern als Unterräume der  $n$ -dimensionalen Sphäre  $S_n$  die in ihr enthaltenen  $(n-1)$ -dimensionalen Gross-Sphären, d.h. jene in ihr enthaltenen  $(n-1)$ -dimensionalen Sphären, welche denselben Durchmesser, wie die  $S_n$  haben, gewählt werden.

Ausser den  $n$ -dimensionalen Räumen von konstanter Krümmung genügen u.a. die  $n$ -dimensionalen GITTER den Bedingungen unseres Satzes.

Auf die Zusammenhänge der Bedingungen des Satzes mit dem Bewegungsbegriff in allgemeinen metrischen Räumen werden wir in der ausführlichen Darstellung dieser Verhältnisse in einer der folgenden Untersuchungen über allgemeine Metrik näher eingehen.



**Physiology.** — *An Experimental Study of the Cerebellar Control of the Vocal Organs.* By LAURETTA BENDER. (Communicated by Prof. G. VAN RIJNBERK and Prof. B. BROUWER).

(Communicated at the meeting of June 25, 1927)

As a result of his comparative anatomical studies of the cerebellum, BOLK (1) was the first to suggest placing a center for laryngeal control in the anterior lobe. VAN RIJNBERK (2) checked BOLK's other anatomical conclusions with physiological experiments showing that lesions in the lobulus simplex (BOLK) disturbed the function of the muscles of the neck and lesions in Crus I of the lobulus ansiformis disturbed the function of muscles of the ipsilateral fore limb. But KATZENSTEIN and ROTHMANN (3) were the first to attempt experiments to prove the presence of the laryngeal center. They reported that operative lesions to the anterior peduncle, or to the lobus anterior (BOLK) and especially to the lobulus centralis in dogs produced disturbances in the movements of the vocal cords in the nature of tremors, jerky movements, incomplete abduction and adduction and especially abduction in two to four movements. The jaws were lax and the tongues flaccid. The dogs did not bark from one to two months and then their bark was high and metallic. Faradic stimulation of the lobus anterior (BOLK) caused a raising of the larynx, contraction of the jaw and adduction of the vocal cords. GRABOWER (4) repeated the work and reported that all the findings were due to the ether anaesthesia or to current spread.

Under the direction of Professor G. VAN RIJNBERK the work was again repeated with a new surgical approach and objective studies of the vocal activities. The anatomical material so obtained was prepared and studied under the supervision of Professor B. BROUWER.

The other workers had approached the anterior lobe by entering the vertex of the skull, lifting the occipital lobes of the cerebrum, and destroying the tentorium. This was, of course, in the neighbourhood of the large sinuses. In this work the approach was made through the base of the skull and the fourth ventricle by elevating the lobus posterior (BOLK) of the cerebellum. Nine dogs survived the operation and were available for further physiological and anatomical studies, seven of which were operated by this procedure and two by a direct approach to the dorsal surface of the cerebellum with injuries inflicted in the culmen and tuber.

Besides direct observation, two methods of objectively recording the vocal cord movements were used. Kimographic records were made by



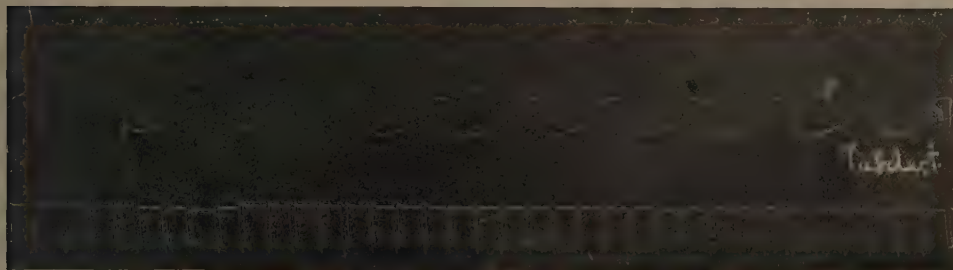
Fig. 1

placing one end of a needle in one of the vocal cords with the other end recording on a drum. Records of the respiratory movements were made with a Marey tambour at the same time. This method proved to be harmless to the vocal cords and under light morphine anaesthesia could be continued for hours. Besides this, cinematographic records were made by taking moving picture of the vocal cord movements (Fig. 1), measuring the distance between the vocal cords in each exposure, and charting one-half of this distance, or the movement of one vocal cord, against time. All dogs were examined before operation, a day or two afterwards, and again as indicated.

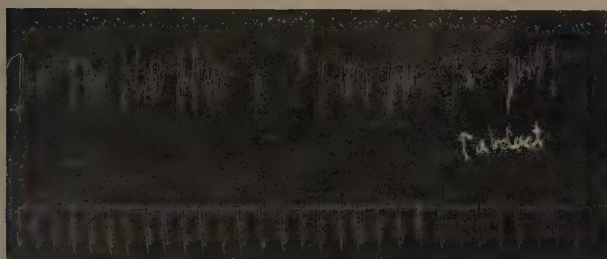
It is necessary to note the age of the dogs, as in very young dogs the movements of the vocal cords are likely to be uncertain and irregular and in very old ones they are likely to be tremulous. But in young adult dogs under light morphine anaesthesia the vocal cord movements occur regularly with respiration as seen in the kimographic record (Fig. 2) and the cinematographic record (Fig. 3). Now if ether is given, there is a marked change in the picture. The movements first increase in rate and irregularity with the respiration during the excitement period. If the anaesthesia becomes very deep, the cords may be held open, the movements gradually returning as the anaesthesia lightens and the shallow respiration becomes deeper. During that stage of anaesthesia in which the reflexes are just absent there occurs a jerking abduction, or abduction in two to three movements. (Figs. 2 and 3). This was described by KATZENSTEIN and ROTHMANN as a result of cerebellar lesions and by GRABOWER as a phenomena of ether anaesthesia.

After operation four of the dogs showed no changes in the vocal cord movements. In the other five, abnormal movements were observed and recorded in the nature of accessory movements occurring in the vocal cords without being reflected from respiratory movements together with general irregularities (Figs. 2a (Plate) and 3). These accessory movements had certain characteristics which made it seem reasonable that they were of cerebellar origin. They were most pronounced during the first few days after operation, then, as the other cerebellar symptoms improved, these accessory movements disappeared, so that by the time of the dog's death on the

LAURETTA BENDER: "AN EXPERIMENTAL STUDY OF THE CEREBELLAR CONTROL OF THE VOCAL ORGANS".

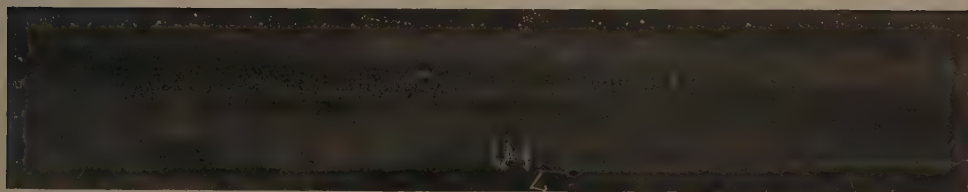


Normal vocal cord movements.

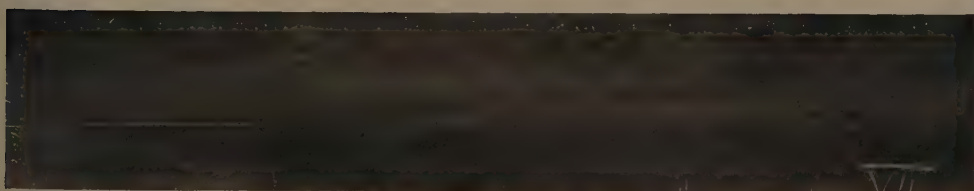


Accessory movements seen in the vocal cord movements with vocalization 24 hours after operation on the cerebellum.

Fig. 2a. Kimographic records of the left vocal cord. Dog 17.



Barking and Ut<sub>4</sub> tuning fork.

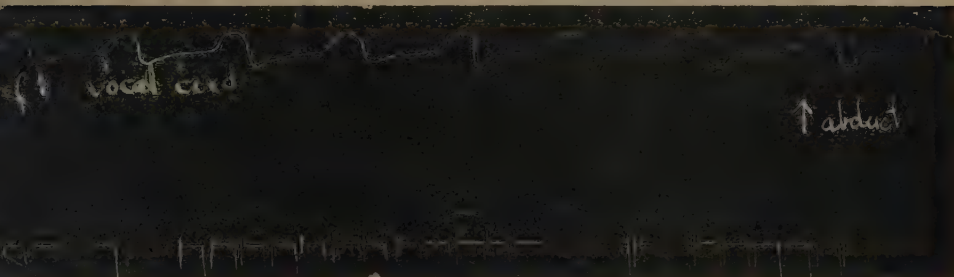


Whining.

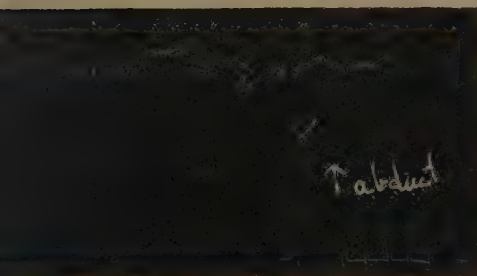
Fig. 4. Normal whining and barking of Dog. 7.



eighteenth day, none could be seen. The accessory movements increased with the vocal activities of the cord. Thus if the dogs attempted to



Vocal cord movements with ether anaesthesia.



Accessory movements without vocalization.

Fig. 2. Kymographic records of the left vocal cord. Dog. 17.

howl under morphine while the cords were being observed, the accessory movements would immediately appear or increase. In this way they may be compared with an action tremor such as was also noted in some of the dogs' head and neck movements when they attempted to take food or water from a pan. Similarly in human speech disturbances of cerebellar origin, the difficulties are said to increase with the patient's effort. Since the vocal cord movements may be looked upon as a form of alternating motion, the disturbances in them may be said to be a form of adiadochokinesis. This has also been pointed out by BONHOEFFER (5). And if the curves obtained in this work are compared with those obtained by HOLMES (6) in his analyses of the motor disturbances following war wound injuries to the cerebellum, they will be seen to be quite similar. Thus the irregularities found in the vocal cord movements compare well with those shown by HOLMES in curves of supination and pronation of the arm and the curves showing the accessory movements of the vocal cords look not dissimilar to the curves of the knee jerks in patients with cerebellar lesions. HOLMES speaks of "adventitious" and "pendular"



movements where the terms "accessory" movements and "irregularities" are used here. And as he says that the disorders in phonation and

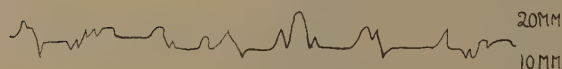
Dog 23 Ether.



Dog 23 Normal



Dog 17 Post-operative.



10 Sec.

20 Sec.

Fig. 3. Cinematographic curves of the dog's vocal cord movements.

articulation found in his patients were due to the same muscular disturbances that were found elsewhere, we might well expect a similarity in the curves.

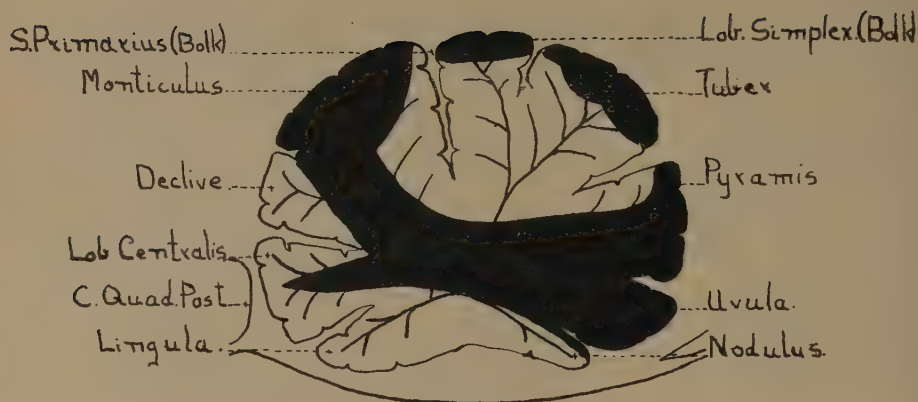
Phonetic studies of the dogs' barking and whining were made with the assistance of Dr. L. KAISER, Lector in Experimental Phonetics of the University of Amsterdam. Records of whining were made under morphine on EDISON Phonograph records and transcribed to smoked paper (Fig. 4, Plate). One dog was trained to bark into the phonograph and similar records made. But these experiments did not show any specific disturbances in the dog's vocal activities after operation. Five of the nine dogs barked normally before their death on the eighteenth day, three died of conditions subsequent to their operation and were never well enough to bark and only one did not bark when it might have been expected to do so and this may have been an individual peculiarity. The fact that they often did not bark for several days is accounted for by the fact that they were very sick, weak, often had forced movements, and were in no mood to bark. They whined and howled normally. The phonetic studies showed no changes in the whining. Barking was

only studied phonetically in one dog, and this dog showed no changes in the vocal cord movements after operation. It barked in the phonograph on the fifth day, post-operative, and thereafter. The first bark was more simple or more like a whine, but this is explained by the dog's weakness. It was quite normal in a few days. Incidentally this dog proved to have a severe lesion in the anterior lobe. It should be emphasized, however, that the absence in any disturbance of barking or whining should not be accepted as a fair criterion for disturbance in vocal cord function of the nature which would be here anticipated. For these activities are relatively simple and the disturbances of cerebellar origin which are reported in human speech are of a much more complex nature in the realm of phonation and articulation.

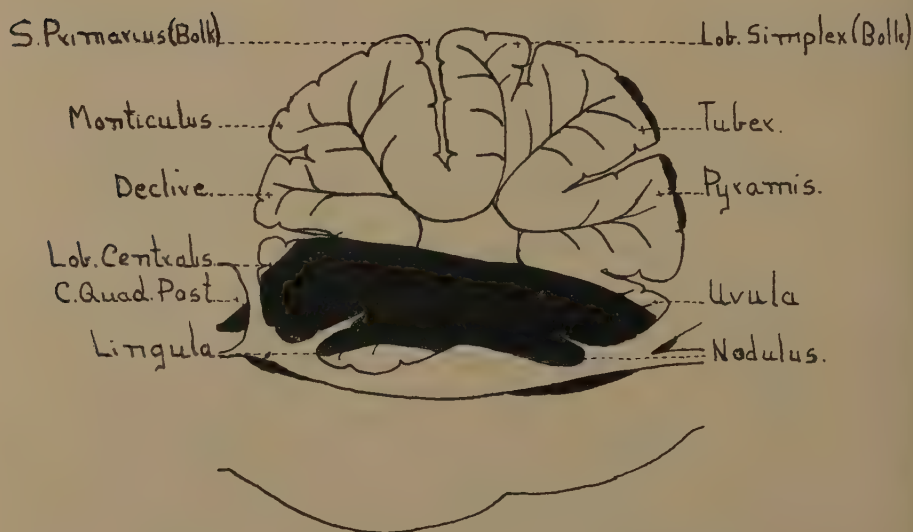
The dogs were allowed to live for eighteen days after the operation, during which time the other cerebellar symptoms were also observed. The brains were prepared by the MARCHI technique with complete serial sections through the whole of the cerebellum and the related parts of the brain stem. The extent of the lesions was determined by the microscope. It was found that the four negative cases included those two in which the dorsal surface of the vermis had been injured superficially, but also two others where the approach had been made through the fourth ventricle in an effort to injure the anterior lobe and the lesion had, in both cases, passed between and partially involved the nuclei fastigii, in one had injured the lobulus centralis, and in the other (the barking dog) had passed through the root of the lobulus centralis to the monticulus. The total range of these four lesions is seen in the composite diagram. (Fig. 5) Parts not directly involved in the primary lesion in the vermis show secondary degeneration of fibre tracts. The important thing in these four brains is, however, that the tissues surrounding the fourth ventricle were entirely protected in every instance by a portion of uninjured cerebellum. From these cases we may say that lesions to all parts of the vermis, including its white tracts and nuclei did not produce any changes in the activities of the vocal cords demonstrable by the methods used.

A composite picture of the range of lesions in the five positive cases shows that their range is not so great as that of the negative group. Furthermore, in one of these dogs the lobulus centralis and all of its fibre tracts and nuclei remained uninjured. But all showed some lesions in the tissues about the fourth ventricle. Those in the floor varied in position and extent, being found in such places as in the fasciculi of GOLL and BURDACH, tissues superficial to the central grey mass, and the medial surfaces of the peduncles as well as the corpora quadrigemina postica. These lesions were usually slight and no one area was involved in all five dogs, two of them showing practically negligible injuries to the most inferior part of the floor of the fourth ventricle.

The only region which was involved in lesion in all five positive



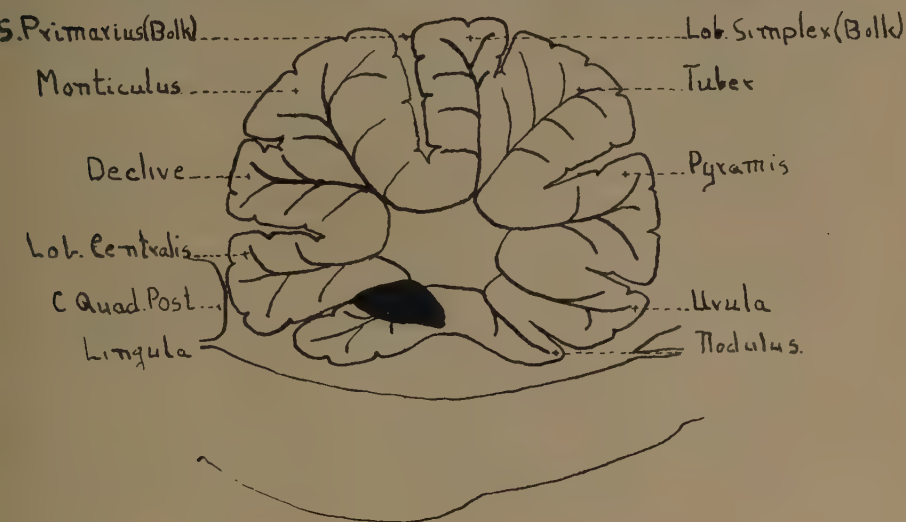
Range of lesion in four negative dogs.



Range of lesion in five positive dogs.

Fig. 5. Schematic diagram of the cerebellum of the dog drawn on the mesial plane showing composite pictures of the lesions.

animals was at the base of the lingula (Fig. 6). And this is the region left uninjured in the four negative animals. Microscopical study of the



Area injured in all five positive dogs.

Fig. 6. Schematic drawing of the cerebellum of the dog on the mesial plane showing area involved in all positive dogs.

sections at this level shows that the tissues involved are not only the lingula on but the neighbouring tissues at the angle formed by the junction of the roof and floor of the fourth ventricle. This is the area called by MEYNERT the „innere Abteilung des Kleinhirnstiels" (AIK) and includes the tractus nucleocerebellaris composed of ascending and descending fibres between the various medullary nuclei and the cerebellum.

We may thus conclude that lesion to any part of the vermis including the roof nuclei has not produced any changes in the activities of the vocal cords but that lesions in those tracts which connect the cerebellum and the medulla do produce definite changes in the motor function.

The question of a localization of an area in the cerebellar cortex for the control of the vocal organs has been under discussion for some time. As has been said BOLK placed it in the lobus anterior (BOLK) with the bilaterally co-ordinated muscles of the face and head on the basis of comparative anatomical studies. But INGVAR (7) argued that the speech function belonged to the youngest, phylogenetically, in the animal series and as such should belong to the youngest part of the cerebellum, or that part just below the sulcus primarius (BOLK). It should be remembered

in this connection, however, that the speech disturbances of cerebellar origin take the form of a dysarthria and not a dysphasia, and although speech is a recently acquired function, the use of the vocal cords in sound production is an old function. JELGERSMA (8) on the basis of an argument that the erect posture, activities of dexterity, and speech have developed along parallel lines in man and that all are dependent on a highly developed muscle sense and motor co-ordination, which functions he places in the cerebellum; and since the hemispheres have developed most in man he places these functions in this part. And since the organs of speech are bilateral, he believes that cerebellar lesions which will produce speech disturbances must be present in both cerebellar hemispheres. BONHOEFFER (5) as a result of his clinical experience holds to the same view. STENVERS (9) from an analysis of his own clinical cases and others in the literature concluded that speech disturbances might arise from unilateral lesions provided the lesion was placed on the right side in righthanded individuals and vice-versa. This idea was refuted by BROUWER (10) who held that the lesion must be bilateral and probably also involve the vermis or paleo-cerebellum. HOLMES (6) as a result of his extensive experience with gun shot wounds in the cerebellum concludes that speech may be disturbed by lesions of any portion of either the vermis or the hemispheres although it is more liable to be severely disturbed when the vermis is injured.

In this series of experiments the hemispheres were not explored. The lobuli paramediani (BOLK) were accidentally injured on their medial surface in some cases but these injuries were slight and also occurred in two of the negative cases. Lesions in one or both of the hemispheres or very extensive lesions of the cerebellar cortex alone might produce the symptoms which were here found only with peduncular lesions. Of course vocalization is much simpler in the dog and volitional vocalization is not easily studied and it may readily be that even extensive lesions in the cerebellar cortex of the dog will produce no symptoms in vocal cord function when such lesions in man would produce definite changes in phonation and articulation. In any case we must conclude that the cerebellum in its connections with the brain stem does appear to have some control over the motor activities of the vocal cords in dogs but that this control does not appear to be localized in any particular part of the cortex so far studied.

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**Geology.** — *The Meteorite of Ellemeet* (after that of Uden in 1840, and that of Blauwkapel in 1843, the third in the Netherlands). By W. NIEUWENKAMP. (Communicated by Prof. L. RUTTEN.)

(Communicated at the meeting of June 25, 1927).

Whit a loud noise, so that horses and cows took fright, this meteorite fell down in a meadow near Ellemeet, in the island of Schouwen (Holland) on the 28<sup>th</sup> of August 1925 about 11<sup>30</sup> in the morning. The only eye-witness describes the phenomenon as "a golden egg with a golden arrow", and asserts to have seen a thin cloud of smoke, 2 m. in length, accompanying the stone. The stone fell almost vertically, slightly from the south, and penetrated into the sods to a depth of about  $\frac{1}{2}$  m., according to the written reports sent by Mr. A. P. VAN DER WEIJDE and Mrs. M. WOLFERT of Ellemeet, to the Meteorological Institute at de Bilt, and to a communication in the "Terneuzensche Courant" of 4 Sept. 1925. This information was afterwards followed by three letters from Mr. W. L. VAN DER LINDE, Headmaster in a neighbouring village. He informed us that about 11 o'clock on the same day, landlabourers heard three bodies coming from the northwest with a whistling, howling sound, as of projectiles flying past, and falling down with a dull thud. Now it is a fact that  $1\frac{1}{2}$ —2 km. northwest of the spot where the first stone was found, a second stone had come down. This was of a dull, grey colour, and weighed about  $\frac{1}{2}$  kg. It had gone into the ground at an angle of about 70° to a depth of about 40 cm. When this piece had been dug up, it was left exposed to the action of weather and wind, and has almost completely been pulverized.

The data concerning these two stones do not quite agree, as the second stone came down to the north-west of the first, and moreover, came from a northwestern direction. One might therefore have expected the first stone also to have come from the north-west. And as stated this stone came from the south. Besides, there is also a difference of half an hour in the time-data.

A third stone was not found.

The first stone has been sent to the Meteorological Institute at De Bilt. While being dug up the stone broke into five pieces weighing about 970 grms. (presumably some of the material had already been lost). The investigation brought on another loss of material, so that 4 of them now weigh 269, 239, 208, and 177 grms. A few smaller fragments are still left, weighing together circa 25 grms. Total 918 grms.

The specific weight of the stone is estimated at 3.41. For this determination the first fragment was weighed under water, after the air had been removed from it by means of an airpump. The form of the stone

must have been approximately spherical (diameter 10 cm.), so far as could be ascertained by fitting the fragments together. On one side the stone is rather smooth, on the other side it is blebby and pitted with irregular "thumb-marks". The crust is dull black, a bit shiny where larger crystals are fused. Fine points of magnetite, which mineral apparently yields but slowly to the corrosion, are sticking out of the crust. A close network of fine suture-like cracks is seen on a large portion of the surface of the crust (meshes from 2 to 5 square mm.).



Something of it can be seen on the top-fragment in the figure. The crust is thin (about 0.1 mm.).

The structure is tuff-like, larger crystals of Hypersthene and a few grains of Olivine are lying in a matrix of Pyroxene-splinters. This matrix is easily friable between thumb and finger.

Veins, Chondres and metallic iron are lacking.

Under the microscope we observed in the slide a very large Olivine-crystal (largest dimensions 1 cm.) with some smaller ones, distinguished by the absence of cleavage and by high double refraction. The axial angle was  $86^\circ$ , optical character negative. Dispersion: red smaller than violet. It contains numerous inclusions of brown glass and grains of ore, the latter being mostly arranged along planes. A systematic position of these planes could not be indicated. I was also able to identify macroscopically a few brown Olivine-crystals, which gelatinized also with hydrochloric acid. Many other larger crystals, which might be taken for Olivine on account of their green transparency, proved, on closer inspection to consist of rhombic Pyroxene. The Pyroxene-crystals present a great variety



exteriorly. Some of them are very dark, probably by numerous inclusions, a large number of which could be recognized under the microscope (again arranged in irregular planes). Others have the appearance of magnetite, as to their colour; it may be they are enveloped by a thin layer of it.

The axial angle of the pyroxenes was estimated at 60–65°, optical character negative.

Furthermore, granules of magnetite and chromite could readily be identified, the latter being brownish red, transparent and isotropic, the former being strongly attracted by a magnet.

Light yellow granules easily soluble in hydrochloric acid, were considered to be pyrrhotite. Some grains were not or only slightly magnetic.

Feldspar was not found.

Prof. N. SCHOORL was so kind as to have determined the chemical composition in the pharmaceutical laboratory:

	Weight %	Molecular %
SiO <sub>2</sub>	53.63	48.6
Cr <sub>2</sub> O <sub>3</sub>	1.82	0.7
FeO	18.95	14.3
MgO	25.65	34.8
MnO	1.35	1.0
S	0.42	0.7.

It is remarkable that this composition demonstrates that Olivine is practically absent, although in the slide it was found in the larger crystals, and in rather large quantities. Mr. G. T. PRIOR of London to whom I sent a fragment of the stone, also detected some Olivine, little though it may be, in a slide cut from the stone. In 3 microscopical sections of two granules of only a few grammes Mr. A. LACROIX of Paris also observed several Olivine grains and considered the resemblance with Roda complete. The fact that the analysis shows the absence of Olivine, would then be accounted for by an inhomogeneity similar to that found by A. LACROIX of Roda <sup>1)</sup>, which led him to the supposition that larger crystals of Pyroxene and Olivine had been consolidated into a breccia. A difference with Roda is the total absence of feldspar, which occurs in Roda, in small quantity.

At any rate in the Brezina-system the Ellemeet belongs to the Rodite, defined as Achondrite, consisting of Rhombic Pyroxene with Olivine, breccia-like. Also in the new nomenclature of A. LACROIX it will be classed under the same group, although according to his definition the Olivine must have been identified under the microscope as well as by the chemical analysis; there is, however, no reason to suppose that a second chemical analysis made of a fragment in which the Olivine has been shown under the microscope, should not demonstrate its presence in the same fragment.

<sup>1)</sup> La Météorite de Roda. Note de A. LACROIX. Comptes Rendus de l'Académie des Sciences Paris. 180, 1925, p. 89.